PHYSICS DEPARTMENT GRADUATE PRELIMINARY EXAMINATION
PRINCETON UNIVERSITY

Monday, January 10, 2000 - 9:00 am - 12:00 noon

Part I.

Answer two out of the three questions in Section A (Mechanics) and two out of the three questions in Section B (Electricity and Magnetism).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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Part I. Section A. Mechanics

1. The shape of an arch is determined by the condition that each brick is held in place by the normal force of its neighbors, with no need for mortar or "glue." To model this consider a thin course of bricks shaped so the normal force exerted on each brick by the neighbor on either side supports the brick against the uniform gravitational acceleration $g$. Then imagine the limit where the arch is a thin line with height $y = y(x)$ as a function of horizontal position $x$. The constant mass per unit length along the line of the arch is $\mu$.

Find $y(x)$ for an arch with horizontal width $2l$ and height $h$. Give the equations that determine the constants in your solution, but you need not solve for the constants.

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Part I. Section A. Mechanics (continued)

2. The centers of two thin rods with mass $m$ and length $l$ are connected by a thread, and the two ends are connected by a very short flexible thread. The threads have negligible mass. The other ends of the rods are free to slide without friction on a horizontal table. The plane of the rods is vertical to the table, so the gravitational acceleration $g$ is in the plane of the rods, as shown in the drawing. The rods are at rest, and make angle $\theta$ with the table. When the thread connecting the centers is cut the rods fall vertically until they hit the table.

a) Find the speed at which the connected ends of the rods are falling immediately before they hit the table.

b) Find the tension in the thread that connects the ends of the rods immediately before the rods hit the table.

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Part I. Section A. Mechanics (continued)

3. A mass $m_1$ slides without friction on a horizontal table. The mass is tied to a string with negligible mass that passes without friction through a small hole. A mass $m_2$ is tied to the other end of the string. The uniform gravitational acceleration $g$ is normal to the table.

The orbit of $m_1$ is only slightly perturbed from circular. The masses $m_1$ and $m_2$ are chosen so the orbit is closed, with one maximum and one minimum of the distance $r(t)$ of $m_1$ from the hole, when computed to first order in the departure from a circular orbit. Find $m_2$ in terms of the other parameters.

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Part I. Section B. Electricity and Magnetism

1. **A Two-Wire Transmission Line**

A transmission line consists of a pair of conducting wires each of radius $a$ whose centers are distance $b$ apart. The space surrounding the wires has unit dielectric contant and permeability. Deduce the capacitance $C$ per unit length.

[From this you could deduce the inductance $L$ per unit length using $LC = 1/c^2$, the impedance $Z = \sqrt{L/C} = 1/cC$, and the sensitivity of the impedance to an error $\delta b$ in the wire spacing, *etc.*]

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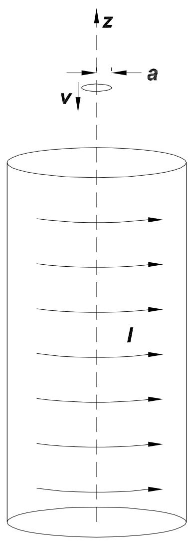
Part I. Section B. Electricity and Magnetism (continued)

2. **Pitching Pennies into a Magnet**

If one pitches a penny into a large magnet, eddy currents are induced in the penny, and their interaction with the magnetic field results in a repulsive force, according to Lenz' law. Estimate the minimum velocity needed for a penny to enter a long, solenoid magnet with central field $B = 1$ T and diameter $D = 0.1$ m.

You may suppose that the "penny" is actually a thin ring (torus) of radius $a$, cross-section area $\pi b^2$ where $b \ll a$, mass density $\rho$ and conductivity $\sigma$. The "penny" moves so that its axis always coincides with that of the magnet, as shown in the figure below. Ignore gravity. The speed of the "penny" is low enough that the magnetic field caused by the eddy currents may be neglected compared to that of the solenoid. Equivalently, you may assume that the magnetic diffusion time is small.



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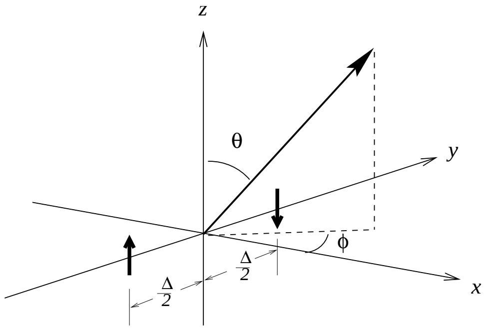
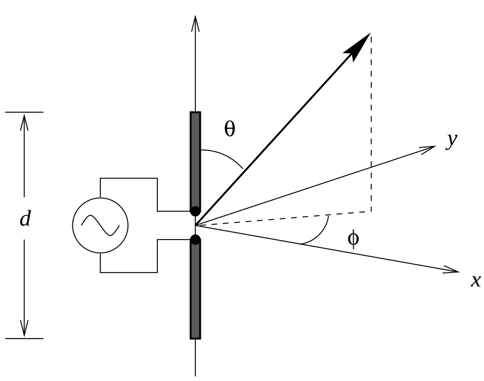
Part I. Section B. Electricity and Magnetism (continued)

3. **A Phased Antenna Array** Two "short" dipole antennas form a small "phased array" as shown in the figure. The second dipole is placed a distance $\Delta = \lambda/2$ away from the first along the $y$ axis. The two dipoles are parallel to one another and are driven 180° out of phase of one another.




Each antenna is a center-fed dipole radiator formed from two wires, each of length $d/2 \ll \lambda$ and driven by a current source as shown in the figure below. The wires are aligned parallel to the $z$ axis ($\theta = (0, \pi)$). The current source produces a time-dependent current given by $I(t) = I_0 e^{-i\omega t}$. You may assume that the charge that enters the wires is uniformly distributed along their lengths.




**This problem is continued on the next page.**

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Part I. Section B. Electricity and Magnetism (continued)

Problem 3. (continued)

Calculate the time-averaged angular distribution of the radiated power for this arrangement in the radiation zone as a function of $\theta$ and $\phi$, *i.e.*, calculate $\langle dP(\theta, \phi)/d\Omega \rangle$.