PHYSICS DEPARTMENT GRADUATE PRELIMINARY EXAMINATION
PRINCETON UNIVERSITY

Tuesday, January 11, 2000 - 9:00 am - 12:00 noon

Part II.

Answer two out of the three questions in Section A (Quantum Mechanics) and two out of the three questions in Section B (Thermodynamics and Statistical Mechanics).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

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Part II. Section A. Quantum Mechanics

1. A particle of mass m and energy $\hbar^2 k^2/2m$ scatters in a central potential $V(r)$ which is everywhere positive and vanishes rapidly as $r \to \infty$. Let $d\sigma/d\Omega$ be the differential cross section as computed in the Born approximation. For precisely backwards scattering you are given

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{back}} = A\frac{\exp(-4\lambda k)}{k^2}$$

where A, λ are given parameters.

a) Calculate $d\sigma/d\Omega$ in the same approximation for arbitrary scattering angle.

b) Calculate $V(r)$.

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Part II. Section A. Quantum Mechanics (continued)

2. Two interacting particles have Hamiltonian $H = H_0 + H'$, where

$$\begin{aligned} H_0 &= -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + V(\vec{r}_1) + V(\vec{r}_2), \\ V(\vec{r}) &= \frac{1}{2}k|\vec{r}|^2, \\ H' &= \epsilon(x_1 x_2 + y_1 y_2 - 2z_1 z_2). \end{aligned}$$

Find the ground state energy to lowest non-vanishing order in ϵ.

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Part II. Section A. Quantum Mechanics (continued)

3. An electron moves on a circular ring of radius R in the $x - y$ plane. A constant uniform magnetic field B is applied in the z direction. Calculate the energy levels. Take the gyromagnetic ratio to be $g_s = 2$.

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Part II. Section B. Thermodynamics and Statistical Mechanics

1. A container C consists of two equal cubes joined together with a thin pipe through which gas can flow. We insert a monatomic, paramagnetic gas in C and it is at equilibrium at temperature T. The spin of each atom is 1/2 and its magnetic moment is $g\mu_B$. The mass of each atom is m.

   Next we place one of the cubes between the poles of a magnet which produces a field, H, in this cube, but not in the other. Assuming that there is no change in temperature and that the gas can be considered to be 'ideal', what is the ratio of the pressures in the two cubes?

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Part II. Section B. Thermodynamics and Statistical Mechanics (continued)

2. An absent-minded physicist has a container of gas, but has forgotten whether it is Argon (Ar) or Nitrogen (N_2).

An experiment is performed using a piston of area A and some small masses which are slowly placed on top of the piston one by one, as shown in the figure. The weight of the piston is negligible. The walls are insulating and the gas is 'ideal'.

The system is initially at rest, without masses on the piston. As the masses are added the height h is measured; the following relation is observed to describe the total masses as a function of piston height:

$$M_0 + M(h) \propto h^{-n},$$

where M_0 is a constant.

If the gas were N_2 what would the exponent n be? What is M_0?

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Part II. Section B. Thermodynamics and Statistical Mechanics (continued)

3. The object of this problem is to find a method for producing Bose-Einstein condensation by means of evaporation. Consider a thermally insulating container of volume V containing N noninteracting particles obeying Bose-Einstein statistics.

   a) Find the equilibrium particle distribution in momentum space in terms of the temperature and chemical potential when the particle number density is less than the critical value $\rho_c(T)$ for Bose-Einstein condensation.

   b) Find the equilibrium particle distribution in momentum space when the particle number density is larger than $\rho_c(T)$.

   c) The critical density $\rho_c(T)$ for Bose-Einstein condensation scales with temperature as T^γ. Use this distribution (plus some elementary scaling) to find γ.

   d) Suppose, at first, that the density ρ is at the critical density $\rho_c(T)$. We allow some of the particles to escape. The few particles that escape have mean energy Ae, where e is the mean energy of the particles that remain and A is some experimentally determined constant. You are allowed to assume that the change in ρ is small. The system, which is thermally insulated, is then allowed to return to equilibrium. The question to be answered is this: For what values of A does the system remain in the condensed phase?