PHYSICS DEPARTMENT PRINCETON UNIVERSITY

GRADUATE PRELIMINARY EXAMINATION

May 4, 2000 - 9:00 am - 12:00 noon

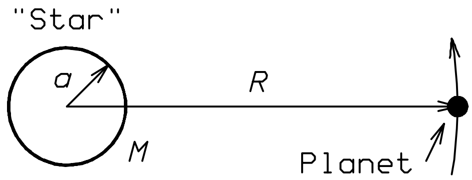
Part I.

Answer two out of the three questions in Section A (Mechanics) and two out of the three questions in Section B (Electricity and Magnetism).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

May 4, 2000 - 9:00 am - 12:00 noon

Part I. Section A. Mechanics



1. With Newtonian mechanics, we wish to compute the rate of precession of the perihelion (point of closest approach) of a planet in orbit around a stationary ring-shaped "star" of radius a and mass M. The planet orbits in the plane of the ring and its distance, R, from the center of the ring satisfies $R \gg a$.

The situation in this problem is a toy model for the effects of an oblate sun. Professor Dicke (and others) pointed out that an oblate sun could be responsible for part of the excess precession of the perihelion of Mercury—an effect usually attributed entirely to general relativity.

a) What is the gravitational potential of the ring in the plane of the ring? Include terms to order $(a/R)^2$.

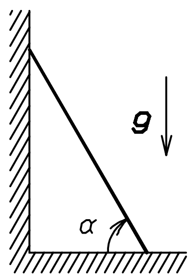
b) What is the angular velocity, ω_0, of a circular orbit of radius R, to order $(a/R)^2$?

c) If the planet is given a small radial perturbation, its new orbit will oscillate about the original circular orbit with angular frequency ω_r. Find an expression for the precession of the perihelion, $\Delta\phi = 2\pi(\omega_r - \omega_0)/\omega_0$.

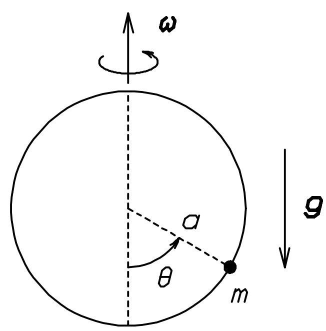
May 4, 2000 - 9:00 am - 12:00 noon

Part I. Section A. Mechanics (continued)

2. A uniform ladder leans against a frictionless vertical wall and rests on a frictionless horizontal floor. It is released from rest, with the ladder and the floor initially making an angle α. At some point, the ladder will separate from the wall. Determine the angle the ladder makes with the floor when this happens.



3. A circular hoop of radius a rotates about a vertical diameter with constant angular velocity $\boldsymbol{\omega}$. A small bead of mass m is constrained to slide without friction on the hoop. Consider the case when $\omega^2 = g/a$. The bead can undergo small oscillations around $\theta = 0$. These are not simple harmonic oscillations! Determine the period of these small oscillations. You may leave an unevaluated definite integral in your expression, but your solution should make it obvious how the period depends on the amplitude of oscillation.



May 4, 2000 - 9:00 am - 12:00 noon

Part I. Section B. Electricity and Magnetism

1. A conductor is at temperature T in a vacuum. The goal of this problem is to deduce the flux density emitted from the conductor at angle θ from the normal to its surface. Recall Kirchhoff's law of heat radiation (as clarified by Planck):

$$\mathcal{E}_\nu = A_\nu K(\nu, T) = A_\nu \frac{h\nu^3/c^2}{e^{h\nu/kT} - 1},$$

where A_ν is the unitless absorption coefficient and $\mathcal{E}_\nu$ is the flux density (power per area per frequency) emitted from a body at temperature T. Specifically, one can write:

$$A_\nu = 1 - R_\nu = 1 - \left|\frac{E_r}{E_i}\right|^2.$$

Also recall Fresnel's equations of reflection:

$$\left.\frac{E_r}{E_i}\right|_\perp = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \qquad \left.\frac{E_r}{E_i}\right|_{||} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)},$$

where i, r, and t label the incident, reflected, and transmitted waves, respectively, and $\perp$ and $||$ refer to the plane of emission.

a) Begin by finding an approximate expression for the complex wave vector k_t as a function of frequency for the wave transmitted into a good conductor ($4\pi\sigma/\epsilon\omega \gg 1$, and $\mu \approx \mu_0$).

b) Find $\mathcal{E}_{\nu_\perp}$, the flux density emitted polarized perpendicular to the plane of emission.

This problem is continued on the next page.

May 4, 2000 - 9:00 am - 12:00 noon

Part I. Section B. Electricity and Magnetism (continued)

Problem 1. (continued)

c) Find $\mathcal{E}_{\nu_\parallel}$, the flux density emitted polarized parallel to the plane of emission.

d) Comment briefly on the polarization of the thermally emitted radiation in the grazing case ($\theta \to 90°$).

2. A particle of mass m and charge q is released from rest from a distance z_0 above an infinite grounded conducting plane. Neglect relativistic effects and gravity.

a) How long will it take for the particle to hit the plane? (Neglect radiation loss.) You may leave your answer in terms of a *dimensionless* integral.

b) What is the power radiated as a function of z?

Now consider the conducting plane to be replaced by a semi-infinite dielectric ϵ. (That is, for $z > 0$, there is a vacuum, and for $z < 0$, space is filled with the dielectric.)

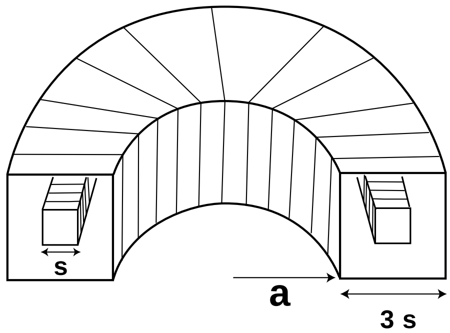
c) Calculate the force on the charge q when it is a distance z_0 above the plane.

Hint: an image solution exists where image charges are placed at either $+z_0$ or $-z_0$.

May 4, 2000 - 9:00 am - 12:00 noon

Part I. Section B. Electricity and Magnetism (continued)

3. The sketch shows two concentric toroidal solenoids. The outer one has N_1 turns of



wire, each with square cross-section with side $3s$. The inner diameter of the outer solenoid is **a**, as shown in the figure. The inner solenoid has N_2 turns of wire, with square cross-section with side s. The resistance of the outer wire is negligible, but the resistance of the inner wire is $\mathcal{R}$.

a) If an AC voltage of amplitude V_0 and angular frequency ω is applied to the outer wire, what is the power dissipated in the system? You may leave your answer in terms of parameters given and the inductances of the system.

b) Find expressions for the relevant inductances.