Department of Physics, Princeton University

Graduate Preliminary Examination
Part I

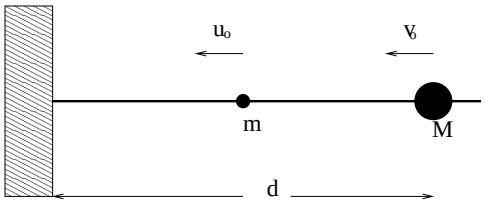
Thursday, May 6, 2004
9:00 am - 12:00 noon

Answer TWO out of the THREE questions in Section A (Mechanics) and TWO out of the THREE questions in Section B (Electricity and Magnetism).

Work each problem in a separate examination booklet. Be sure to label each booklet with your name, the section name, and the problem number.

Section A. Mechanics

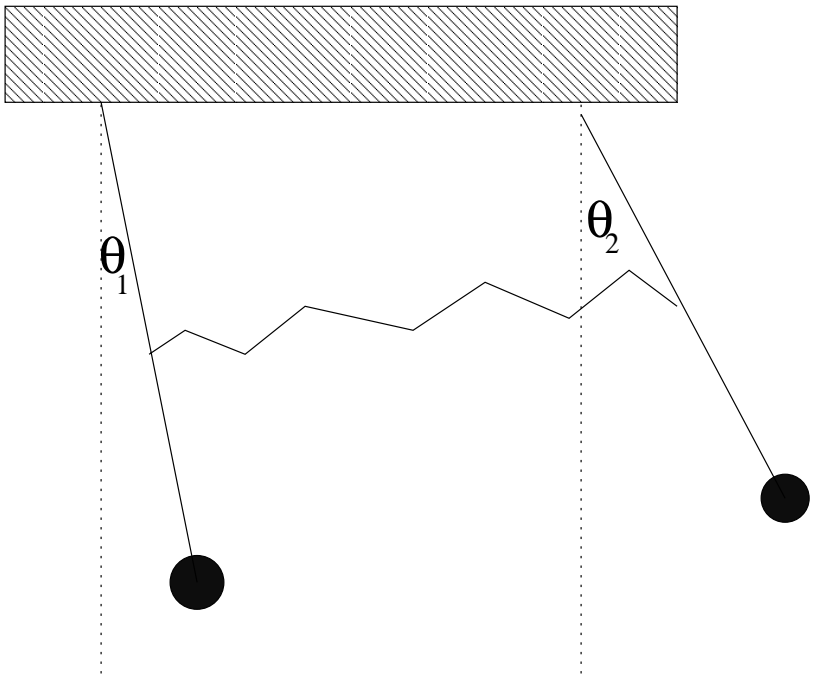
1. Two elastic spherical particles with masses m and M ($m \ll M$) are constrained to move along a straight line with an elastically reflecting wall at its end. At $t = 0$ they are set in motion as shown, with $u_0 \gg v_0$.



Find the subsequent motion of M, averaged over the period of motion of m. In this approximation, how far does M travel before turning around?

(Note: The initial location of m is irrelevant in this limit.)

2. Two simple pendula, each of length l and mass m are coupled by a spring of force constant k which is attached to their massless and inextensible rods at their halfway points. The spring is relaxed when the pendula are vertical.



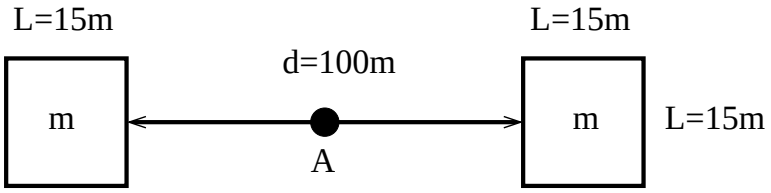
(a) Write a Lagrangian for the system.

(b) Find the normal modes and their frequencies for small oscillations about equilibrium.

(c) At $t = 0$ the left pendulum is displaced by a small angle $\theta_1(0) = \theta_0$ and released from rest while the right pendulum is at rest with $\theta_2(0) = 0$. Find $\theta_{1,2}(t)$.

(d) How long will it be before the energy of the left pendulum is transferred completely to the right pendulum?

3. A set of two rigidly linked panels is placed in outer space, where it can spin about 3 different axes. The panels are in the form of two thin, square, planar sections, $L = 15$ m on a side, each of mass $m = 3 \times 104$ kg. The two square sections are at a distance of $d = 100$ m and are connected with low-mass rigid beams (not shown in the figure).



(a) Compute the principal moments of inertia $I_1 > I_2 > I_3$ about the point A.

(b) After its construction, the set of panels was set spinning about the axis with the moment of inertia I_2, with the angular velocity chosen so that the pseudo-gravity at the center of each square section is $g/6$. Sadly, a tiny asteroid came by soon after and its impact nudged the angular velocity a little bit away from the "2-nd axis". Show that the resulting motion of the panels will be perturbed strongly. What is the characteristic time for the perturbation to grow strong?

Section B. Electricity and Magnetism

1. An uncharged conducting sphere of radius a is coated with a thick insulating shell of dielectric constant ϵ_r, out to radius b. The object is placed in an initially uniform electric field E_0. What is the electric field everywhere in the insulator?

2. In a naive classical model of the hydrogen atom's ground state, the electron moves in a circular orbit of radius $r_0 = 0.53 \times 10^{-10}$ m around the center of mass of the electron-proton pair. Since the electron is accelerating, classically it will continuously radiate energy. Calculate the time it will take the electron to spiral into the proton, assuming its orbit is always nearly circular, and that the motion continues until the radius of the electron's orbit is reduced to that of the proton, 10^{-15} m. Are relativistic effects important for this estimate?

3. A penny is thrown towards a large solenoid magnet. The penny moves along the axis of the solenoid with a frictionless constraint which keeps the plane of the penny perpendicular to the solenoid axis. As the penny approaches the solenoid, eddy currents are induced in it and result in a repulsive force which slows its motion. Estimate the minimal initial velocity which is needed in order for the penny to reach the entrance of the long solenoid magnet, with internal field of $B = 1$ T and diameter $D = 0.1$ m. You may ignore gravity and take the mass density and resistivity of copper to be $\rho \approx 104 kg/m3$ and $\rho_R \approx 10^{-6} ohm.m$, respectively.