Nonlinear, Strong-Field QED
SLAC Experiment E-144

Gil Eisner, *Photonics Spectra, Nov. 1997*

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DESY Workshop

*Probing strong-field QED in electron-photon interactions*

D. Pugh, *Science 277, 1202 (1997)*

http://physics.princeton.edu/~mcdonald/examples/mcdonald_180822.pptx
Abstract
When a powerful laser beam is focused on a free electron the acceleration of the latter is so violent that the interaction is nonlinear.

We review the prospects for experimental studies of nonlinear electrodynamics of a single electron, with emphasis on the most accessible effect, non-linear Thomson scattering. We also speculate on the possibility of laboratory studies of a novel effect related to the Hawking radiation of a black hole.

From the Introduction:
An important qualitative feature distinguishes our understanding of the electromagnetic interaction from that of the strong, weak and gravitational interactions. Namely that the latter are fundamentally non-linear, meaning that the bosonic quanta which mediate these interactions can couple to themselves.

As a step towards the elucidation of fundamental non-linear phenomena we consider the unusual case of very strong electromagnetic fields, in which non-linear effects can be induced.

An appropriate point of departure is the work of Hawking, in which he associated a temperature $T$ with a black hole:

$$T = \frac{\hbar g}{2 \pi ck}.$$ 

Here, $\hbar$ is Planck's (reduced) constant, $g$ is the acceleration due to gravity measured by an observer at rest with respect to the black hole, $c$ is the speed of light in vacuum, and $k$ is Boltzmann's constant.

The significance of this temperature is that the observer will consider himself to be in a bath of black-body radiation of characteristic temperature $T$.

This is in some way due to the effect of the gravitational field on the ordinarily unobservable zero-point energy structure of the vacuum.

*S.W. Hawking, Nature 248, 30 (1974)*

The (Fulling-Davies-)Unruh Effect

Contemporaneous with the work of Hawking, several people considered quantum field theory according to accelerated observers. By the equivalence principle, we might expect accelerated observers to experience much the same thermal bath as Hawking's observer at rest near a black hole.

The efforts of Fulling, Davies, and Unruh indicate that this may well be so.

If \( a^* \) is the acceleration as measured in the instantaneous rest frame of an observer, then (s)he is surrounded by an apparent bath of radiation of temperature,

\[ T = \frac{\hbar a^*}{2\pi c k} \]

\[ S.A. \ Fulling, \ Phys. \ Rev. \ D \ 7, \ 2850 \ (1973) \]
\[ P.C.W. \ Davies, \ J. \ Phys. \ A \ 8, \ 609 \ (1975) \]
\[ W.G. \ Unruh, \ Phys. \ Rev \ D \ 14, \ 870 \ (1976) \]

Fulling's contribution (prior to Hawking's) was that the notion of a "particle" can be different for accelerated and inertial observers.

The "vacuum" can appear to contain no "particles" to an inertial observer, but can contain them according to an accelerated observer.
Of experimental interest is the case when the observer is an electron. Then, the electron could scatter off the bath of radiation, producing photons which could be detected by inertial observers in the laboratory. This new form of radiation, which we call Unruh radiation, is (hopefully) to be distinguished from the ordinary (Larmor) radiation of an accelerated electron.

In particular, the intensity of radiation in the thermal bath varies as $T^4$. Hence, we expect the intensity of the Unruh radiation to vary as $T^4 \propto a^{*4}$.

$$\frac{dU_{\text{Unruh}}}{dt \, dv \, d\text{Area}} \approx \frac{dU_{\text{thermal}}}{dt \, dv \, d\text{Area}} \frac{8\pi}{c^2} \frac{h\nu^3}{e^{h\nu/kT} - 1} \frac{8\pi r_0^2}{3} , \quad \frac{dU_{\text{Unruh}}}{dt} \approx \frac{8\pi^3 \hbar r_0^2}{45c^2} \left( \frac{kT}{\hbar} \right)^4 = \frac{\hbar r_0^2 a^{*4}}{90\pi c^6}.$$ 

This contrasts with the $a^{*2}$ dependence of the intensity of Larmor radiation,

$$\frac{dU_{\text{Larmor}}}{dt} = \frac{2e^2 a^{*2}}{3c^3} = \frac{60\pi}{\alpha} \left( \frac{E_{\text{crit}}}{E^*} \right)^2 \frac{dU_{\text{Unruh}}}{dt} = \left( \frac{160E_{\text{crit}}}{E^*} \right)^2 \frac{dU_{\text{Unruh}}}{dt} , \quad r_0 = \frac{e^2}{mc^2}, \quad \alpha = \frac{e^2}{\hbar c},$$

where $E_{\text{crit}} = \frac{m^2 c^3}{e\hbar}$ is the QED critical field strength, and $a^* = \frac{eE^*}{m}$ is the acceleration.

The QED critical field was introduced by Sauter (1931), following suggestions of Bohr and Heisenberg as to the resolution of Klein’s paradox, i.e., the production of $e^+e^-$ pairs in a strong electric field.

O. Klein, Z. Phys. 53, 127 (1929)
F. Sauter, Z. Phys. 69, 742 (1931)
The Unruh Effect

The Unruh radiation effect is a QED correction to Larmor radiation, and is not likely to be directly observed. [For a more optimistic view, see Cozzella et al., *Phys. Rev. Lett.* **118**, 161102 (2017).]

Strictly, the Unruh effect, in which an accelerated observer detects a blackbody spectrum in what an inertial observers considers to be empty space, holds only for uniform, linear acceleration.

However, the Unruh effect has been observed in the broader sense of excitations, by a quasithermal virtual-photon bath, of electrons in storage rings related to their centripetal acceleration.

Bell and Leinaas noted the Unruh effect gives a qualitative explanation of the limit of 92% of the transverse polarization of electrons in storage rings, ordinarily considered as due to quantum fluctuations in their synchrotron radiation.  


Furthermore, the effect of synchrotron-radiation damping rings in reducing the phase volume of an electron bunch is limited by quantum fluctuations, which correspond to an effective temperature, \( T \approx \hbar a^*/2\pi c k \), that excites transverse and longitudinal oscillations (beam height, width, and energy spread).  

KTM, *PAC87*, p. 1196

For the Unruh effect on damping of transverse oscillations in a linear focusing channel, see  

Strong-Field QED

For high acceleration, need strong electromagnetic fields.

The strongest macroscopic electromagnetic fields on Earth are in lasers.

Nonperturbative, strong-field QED is described by two dimensionless measures, \( \eta \) and \( \Upsilon \), of the strength of a plane wave with angular frequency \( \omega_0 \), and 4-potential \( A_\mu = (0, A) \).

1. \[
\eta = \frac{e \sqrt{\left\langle A_\mu A_\mu^* \right\rangle}}{mc^2} = \frac{e E_{\text{rms}}}{m \omega_0 c} = \frac{e E_{\text{rms}} \hat{\lambda}_0}{mc^2},
\]

[\( \eta \) is also defined for a periodic, static magnetic field = “wiggler“.]

\( \eta \) governs the importance of multiple photons in the initial state, and characterizes the “mass shift”, \( \bar{m} = m \sqrt{1+\eta^2} \), of a (quasi)electron in a “background“ electromagnetic wave (laser beam).


2. \[
\Upsilon = \frac{\sqrt{\left\langle \left( F_{\mu\nu} P_\nu \right)^2 \right\rangle}}{mc^2 E_{\text{crit}}} = \frac{2 p_0}{mc^2} \frac{E_{\text{rms}}}{E_{\text{crit}}} = \frac{2 p_0}{mc^2} \frac{\hat{\lambda}_C}{\hat{\lambda}_0} \eta, \quad E_{\text{crit}} = \frac{m^2 c^3}{e \hbar} = \frac{mc^2}{e \hat{\lambda}_C} = 1.3 \times 10^{16} \text{ V cm},
\]

\( \Upsilon \) governs the rate of “spontaneous” \( e^+ e^- \) pair creation (“sparking the vacuum”) in a wave probed by an electron with 4-momentum \( p_\mu = (p_0, P) \).
Where to Find Critical Fields

The magnetic field at the surface of a neutron star (magnetar) can exceed the critical field $B_{\text{crit}} = 4.4 \times 10^{13} \text{ Gauss}$.  

During heavy-ion collisions where $Z_{\text{total}} = 2Z > 1/\alpha$, the critical field can be exceeded, and $e^+e^-$ production is expected, 

$$Z_{\text{max}} \approx \frac{2Ze}{\lambda_C^2} > 2Z\alpha E_{\text{crit}}.$$  


The Earth’s magnetic field appears to be of critical strength as seen by a cosmic-ray electron with $> 10^{19} \text{ eV}$. I.Y. Pomeranchuk, J. Phys. USSR 2, 65 (1940)

The electric field of a bunch at a future $e^+e^-$ linear collider approaches the critical field in the frame of the oncoming bunch $\Rightarrow$ “Beamstrahlung” limit, (but quantum suppression of classical radiation rate.)


The electric field of a focused teraWatt laser appears critical to a counterpropagating 50-GeV electron $\Rightarrow$ SLAC Experiment E-144.
SLAC Experiment E-144

Performed in the Final Focus Test Beam, SLAC (1996-1997)


Terawatt laser system: Chirped-Pulse Amplification (CPA) in a "slab" amplifier. (Special thanks to D. Meyerhofer.)


1 Joule, 1 ps
Nonlinear Compton Scattering

\[ e + n\omega_0 \rightarrow e' + \gamma \]


Rate (@ order \( n \)) \( \propto I^{n-1} \), when normalized to total scattered photon rate.

Process is still roughly describable as perturbative, though entering the nonperturbative regime.

Theory based on Volkov states of a (dressed) Dirac electron in a plane wave.


Two-step process: \( e + \omega_0 \rightarrow e' + \omega \), then \( \omega + n\omega_0 \rightarrow e^+e^- \).

106 ± 14 signal positrons, 22,000 laser pulses. (Inelastic light-by-light scattering)

\[ Rate \propto \eta^{2n}, \text{ where} \]
\[ n = 5.1 \pm 0.2 \, \text{(stat.)} \pm 0.7 \, \text{(syst.)} \]
\[ \Rightarrow 5 \, \text{laser photons} \]

Process is below threshold for 1 photon.
Process is still roughly describable as perturbative, though entering the nonperturbative regime.

Nonlinear (multiphoton) version of Breit-Wheeler scattering, \( \omega_1 + \omega_2 \rightarrow e^+e^- \).


Trident = \( e + n\omega_0 \rightarrow e' + e^+e^- \).

Strong-Field Pair Creation as Barrier Penetration

For a virtual $e^+e^-$ pair to materialize in a field $E$, the electron and positron must separate by distance $d$ sufficient to extract energy $2mc^2$ from the field:

$$eEd \geq 2mc^2.$$  

The probability $P$ of a separation $d$ arising as a quantum fluctuation is related to penetration through a barrier of thickness $d$:

$$P \propto \exp\left(-\frac{d}{\lambda_c}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{\text{crit}}}{E}\right) = \exp\left(-\frac{2}{\gamma}\right).$$

F. Sauter, *Z. Phys.* 69, 742 (1931)


$$Rate_{e^+} \propto \exp\{[-1.8 \pm 0.2 \text{ (stat.)} \pm 0.2 \text{ (syst.)}]/\gamma\}.$$
E-144 Was Ahead of Its Time

Reaching for the Brightest Light (2018)
US Natl. Acad. Sci., p. 15
The technology now exists to have laser fields with $\gamma \gg 1$ when probed by GeV electrons.

Is new physics accessible in this regime?

A difficulty is that the interaction length of an electron in a strong wave field is about

$$l_{\text{int}} \approx \frac{\lambda_0}{\alpha \eta^2} \approx \frac{\lambda_0}{\alpha \gamma^2},$$

for optical fields and GeV electrons.

Hence, for $\gamma \geq 12$, an electron will scatter in less than one wavelength, losing energy, reducing its $\gamma$ value (and leading to an electromagnetic “shower”). [Beamstrahlung!]

That is, high-energy electrons have low probability to reach a region of $\gamma \gg 1$, and the physics of the few unscattered electrons that reach such a region will have a huge background associated with the majority of electrons that scattered on the way in.

There is some interest in the details of the electromagnetic cascades of electron in strong wave fields.


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$E_{\text{max}} \approx E_{\text{crit}} / \theta$ for a laser beam focused to angle $\theta$, above which “sparkling the vacuum”.

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Induced Light-by-Light Scattering Experiment

\( \omega_1 + \omega_2 \rightarrow \omega_3 + \omega_4, \) where final-state photon \( \omega_3 \) goes into an intense laser beam, \( \Rightarrow \) amplitude gets boson enhancement.

Abstract

Past suggestions for a demonstration of light-by-light scattering by a variation of four-wave mixing may be realizable in the near future with tabletop teraWatt lasers.

Introduction

During the Workshop Alexander Varfolomeev pointed out that light-by-light scattering at optical frequencies can be enhanced by use of a third laser beam [1]. Norman Kroll remarked that he had also considered this in the 60’s [2]. Here we consider whether a 1-teraWatt laser, such as that built at U. Rochester for SLAC E-144 [3], could be used to perform such an experiment, and conclude that rates are still somewhat low. Perhaps with the recently reported 50-teraWatt lasers [4] the signal can be seen.


How Could One Have Missed This?

Vacuum should have less than 1 atom in the laser focal volume, ⇒ Separation of atoms > 10 μm ⇒ Pressure < 10⁻⁹ mm.
Usual Light-by-Light Scattering \[ \omega_1 + \omega_2 \rightarrow \omega_3 + \omega_4 \]

The cross section for light-by-light scattering is very small [5]:

\[
\sigma = \frac{973}{10125\pi} \alpha^2 r_e^2 \left( \frac{\hbar \omega}{mc^2} \right)^6 \approx 0.03 \alpha^2 r_e^2 \left( \frac{\hbar \omega}{mc^2} \right)^6 \approx 7.4 \times 10^{-66} \text{cm}^2 \left( \frac{\hbar \omega}{1 \text{ eV}} \right)^6,
\]

where \( \omega \) is the frequency of the incident photons in the cm frame, and \( r_e = e^2/mc^2 \) is the classical electron radius.

For example, suppose we collide two laser beams of \( N \) photons each at right angles (as would be convenient for the 3-beam experiment discussed below) after focusing them to a spot size of order \( \lambda \), the laser wavelength. If the laser pulsewidth is \( \tau \) seconds then the only a fraction \( \lambda/c\tau \) of the photons in each beam occupies the interaction volume (\( \approx \lambda^3 \)) at any moment. We may regard the scattering as consisting of \( c\tau/\lambda \) successive experiments in which \( N\lambda/c\tau \) photons from each beam interact with each other. The total scattering rate would then be

\[
\text{Rate} \approx \frac{c\tau}{\lambda} \left( \frac{N\lambda}{c\tau} \right)^2 \frac{\sigma}{\lambda^2} = \frac{N^2 \sigma}{\lambda c\tau}.
\]

For example, if we have 1 Joule of photons of 1-eV energy (\( \lambda = 10^{-4} \text{ cm} \)) with a pulse length of 1 psec (as for the present Rochester T³ laser), then the rate is only about \( 10^{-24} \) per pulse!
Four-Wave Mixing

The observation of Kroll and Varfolomeev is that when a third laser beam is present and aligned along the direction of a possible final-state photon, the scattering rate is enhanced by the number of photons in the third beam (during each of the subexperiments described above). That is, for \( N \) photons in each of the three beams,

\[
\text{Rate} \approx \frac{c\tau}{\lambda} \left( \frac{N\lambda}{c\tau} \right)^3 \frac{\sigma}{\lambda^2} = \frac{N^3\sigma}{(c\tau)^2}.
\]

On using the above expression for the cross section, we arrive at the form of Kroll:

\[
\text{Rate} = \Gamma \alpha^4 \frac{\lambda_C^5}{\lambda^3(c\tau)^2} \left( \frac{\mathcal{E}}{mc^2} \right)^3 \approx 10^{-6} \frac{(\mathcal{E}[\text{Joules}])^3}{(\tau[\text{psec}])^2},
\]

where \( \mathcal{E} = N\hbar\omega \) is the pulse energy, \( \lambda_C = \hbar/mc \) is the Compton wavelength of the electron, and the numerical factor \( \Gamma \) is roughly \( \pi \) when the spot size is \( \approx \lambda \).

To reach a rate of one scatter per pulse, we would need, for example, 10 Joules in each beam, whose pulselengths have been compressed to 30 fsec.
Configuration with $\omega_4$ Different from $\omega_1 = \omega_2 = 2\omega_3$

![Diagram of laser beams]

Figure 1: Possible arrangement of the laser beams for the induced light-by-light scattering experiment. Beams 3 and 4 are in the plane perpendicular to the plane of beams 1 and 2 that contains the bisector of the angle $2\theta$ between beams 1 and 2.

A convenient configuration is that $\theta_3 = 90^\circ$, which holds when $\sin \theta = \sqrt{\omega_3/\omega_1}$. In our example where $\omega_3/\omega_1 = 1/2$ we require that $\theta = 45^\circ$, so the angle between beams 1 and 2 is $90^\circ$, as mentioned above. Finally, $\sin \theta_4 = \omega_3/\omega_4 = 1/3$, or $\theta_4 = 19.5^\circ$. The angular separation between photon 4 and the other 3 beams is nearly maximal.
The E-144 Collaboration

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Loose Ends in Strong-Field QED

Workshop on Quantum Aspects of Beam Physics (Monterey, CA, Jan. 4-9, 1998)

Loose End #1: Trident Production. \( e + n\omega_0 \rightarrow e^+ e^- \).

Loose End #2: Strong-Field Light-by-Light Scattering (photon splitting by a laser) \( n\omega_0 + \omega_1 \rightarrow \omega_2 + \omega_3 \)
Loose End #2a: Vacuum Cerenkov Radiation (of an electron in a strong laser beam).

Loose End #3: QED Phase Transition. (Does it exist?)

Loose End #4: From Hawking to Unruh to ...

Loose End #5: Vacuum Laser Acceleration.
Puzzle #1: A charge is accelerated in the E field of a capacitor. Where does the energy come from?
Puzzle #2: A weak plane wave shakes a free electron transverse to the wave (momentum) vector; how is momentum conserved? staticaccel.pdf (1998), transmom2.pdf (1998)
Puzzle #3: A free electron cannot absorb real photons unless it (a) radiates, or (b) changes its mass. So how could a pulse of real photons add energy to a free electron?
Puzzle #4: Compton scattering by a two-frequency wave. \( n_1 \omega_1 + n_2 \omega_1 + n_3 (\omega_1 + \omega_1) + n_4 (\omega_1 - \omega_2) + e \rightarrow e^+ \omega \)
Puzzle #5: An electron inside a (plane, monochromatic) wave is described by the Volkov solutions to the Dirac equation as having a quasimomentum.

Puzzle #5a: If \( \eta \) is large, the threshold is raised for pair creation by light because the final electrons and positrons must be born with mass \( \bar{m} = m \sqrt{1+\eta^2} \), not mass \( m \).

Puzzle #5b: Is neutron decay suppressed in a strong wave field, where \( \bar{m} - m \) could be much larger than the neutron-proton mass difference?
Possible scheme for the positron source at a future linear collider.

Demonstrated 80% longitudinal polarization of both $e^+$ and $e^-$. 

