1. (4 points) The electric field $\vec{E}$ across a parallel plate capacitor is increasing to the right with time according to

$$\vec{E} = \vec{E}_0 \frac{t}{\tau}$$

where $\vec{E}_0$ and $\tau$ are constants. The capacitor plates have radius $R$ and are a distance $d$ apart. What is the magnitude and direction of the magnetic field at a point P shown in the picture to the right, a distance $r < R$ from the center of the plates? (Assume $d \ll R$, so edge effects can be neglected.)

Construct an Ampère's loop through point P as shown to the right.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

where $\phi_E$ is the flux through your loop.

$$B(2\pi r) = \mu_0 \varepsilon_0 \pi^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 r^2 E_0}{2\pi \tau}$$

$$B = \frac{\mu_0 \varepsilon_0 rE_0}{2\tau}$$

(Note that $R$ and $d$ don’t matter.)

Magnetic field is out of the page, by right hand rule, since current is in the $+x$ direction.

**Magnitude:** $B = \frac{\mu_0 \varepsilon_0 rE_0}{2\tau}$

**Direction:** positive $z$

Rewrite and sign the Honor Pledge: I pledge my honor that I have not violated the Honor Code during this examination.
2. (a) (3 points) A radio station in Mahwah, NJ, radiates an average power of $P_0$. Assuming the power radiates symmetrically in all directions (spherically symmetrically, including up and into the ground), what is the amplitude $B_0$ of the magnetic field in Princeton, a distance $x$ away from the station?

$$ I = \frac{P_0}{4\pi x^2} $$

$$ I = \left| \frac{E \overline{B}}{\mu_0} \right|_{\text{avg}} = \left( \frac{cB^2}{\mu_0} \right)_{\text{avg}} = \left( \frac{cB_0^2 \cos^2(\omega t)}{\mu_0} \right)_{\text{avg}} = \frac{cB_0^2}{2\mu_0} $$

$$ \frac{P_0}{4\pi x^2} = \frac{cB_0^2}{2\mu_0} $$

$$ B_0 = \sqrt{\frac{\mu_0 P_0}{2\pi x^2}} $$

(b) (2 points) The radio station broadcasts using an electric dipole antenna (like the one shown in lecture) oriented vertically. Your radio uses a loop antenna to detect the radiation. Circle one choice to show which orientation shown maximizes the magnetic flux through the loop. (The loops are the same size, just different orientations.)

(E-field is along y axis, so B-field is along z axis. Maximum flux through loop in choice A)

(c) (1 point) The magnitude of the magnetic field at the location of your loop antenna is given by $B(t) = B_0 \cos(\omega t)$. The loop has an area $A$. What is the amplitude $V_0$ of the voltage induced in the loop?

$$ V = -\frac{d\phi_m}{dt} = -A \frac{dB}{dt} = AB_0 \omega \sin(\omega t) $$

$$ V = V_0 \sin(\omega t) $$

where

$$ V_0 = AB_0 \omega $$