1. A particle of mass \( m \) is inside a cube of side \( L \). (This is a 3-dimensional infinite square well with potential energy equal to zero inside the cube and infinite outside.)

a) (2 pts) Give an expression for the energy levels in terms of the 3 quantum numbers \( n_1, n_2, \) and \( n_3 \).

\[
\psi(x,y,z) \text{ must go to zero at all walls:}
\]

\[
\psi = A \sin(k_1 x) \sin(k_2 y) \sin(k_3 z)
\]

with \( k_{n_1} = \frac{n_1 \pi}{L} \)

\[
E = \frac{k^2}{2m} (k_1^2 + k_2^2 + k_3^2) = \frac{n_1^2 \pi^2}{2mL^2} (n_1^2 + n_2^2 + n_3^2)
\]

\[ E \text{ of 1-dim. box} \]

b) (2 pts) What is the degeneracy of (that is, number of different states in) the ground energy level? of the first excited level? Answer both questions!

<table>
<thead>
<tr>
<th>Ground Level:</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_3 )</th>
<th>Degeneracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Excited:</td>
<td>( 1, 1, 1 )</td>
<td>( 1, 1, 2 )</td>
<td>( 1, 2, 1 )</td>
<td>( 1, 2, 2 )</td>
</tr>
</tbody>
</table>

Now TEN identical particles are put in the same box. The particles are fermions and do not interact with each other. Each particle has two spin states (like an electron).

\[ \text{2 particles/state (sum)} \]

\[ \begin{array}{ccc}
\text{Particles/State} & E & E/Level \\
11 & 3E_1 & 2 \\
12 & 3E_1 & 6 \\
21 & 9E_1 & 2 \\
22 & 60E_1 & 60 \frac{h^2}{2mL^2} \\
\end{array} \]

Problem 2 on back

Rewrite and sign the Honor Pledge: I pledge my honor that I have not violated the Honor Code during this examination.

Signature
2. The ground state wave function of an electron in a hydrogen atom is

\[ \psi_{1s} = Ce^{-\frac{r}{a_0}} \]

where \( C \) and \( a_0 \) are constants. Show your reasoning or a calculation for each part.

a) (2 pts) Consider a small, constant volume \( \Delta V \). At what location (specify \( x, y, z \)) or \((r, \theta, \phi)\) can this volume be placed so that the electron has the highest probability of being found in it?

\[ P_{\text{prob of particle in } \Delta V} = |\psi|^2 \Delta V = \frac{C^2}{\text{const}} (4\pi r^2) e^{-2r/a_0} \]

**Maximum at Origin, \( r = 0 \) or \((0,0,0)\)**

b) (2 pts) Now, consider a small, constant radial interval \( \Delta r \). At what radius \( r \) is the probability for finding the electron between \( r \) and \( r + \Delta r \) the highest?

For fixed \( \Delta r \), \( \Delta V = 4\pi r^2 \Delta r \), the volume of a thin spherical shell. Note that \( \Delta V \) depends on \( r \).

\[ P_{\text{prob of particle in } \Delta V} = |\psi|^2 \Delta V = \frac{C^2}{\text{const}} 4\pi r^2 e^{-2r/a_0} \]

Find Max of \( r^2 e^{-2r/a_0} \):

\[ \frac{d}{dr} (r^2 e^{-2r/a_0}) = 2r e^{-2r/a_0} + r^2 (\frac{2}{a_0}) e^{-2r/a_0} = 0 \]

\[ 2r - \frac{2r^3}{a_0} = 0 \]

\[ \frac{r}{a_0} = 1 \] \[ \text{Max at } r = a_0. \]