PRINCIPLES AND APPLICATIONS OF MUON COOLING

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Summary

The basic principles of the application of "ionization cooling" to obtain high phase-space density muon beams are described, and its limitations are outlined. Sample cooling scenarios are presented. Applications of cooled muon beams in high-energy accelerators are suggested; high-luminosity μ^+-μ^- and μ-p colliders at $\gtrsim$1-TeV energy are possible.

Introduction

Electron-positron (e^+-e^-) colliders have been essential tools in gaining an understanding of particle physics. However, their future use at higher energies is severely limited by radiation processes. Synchrotron radiation in storage rings causes electrons to lose energy at a rate proportional to the fourth power of the electron energy, and this radiation effectively prevents construction of $e^\pm$ storage rings at energies greater than 100 GeV (LEP). e^+-e^- linear colliders are proposed to circumvent this problem. However, they have substantial practical difficulties in obtaining adequate luminosity, are very expensive, and also have particle radiation problems that prevent practical implementation at particle energies $\gtrsim$300 GeV. Another approach, $\bar{p}p$ and pp colliders, can indeed reach multi-TeV energies, but hadron-hadron interactions lack the simplicity of lepton-lepton collisions; lepton-lepton and lepton-hadron colliders are necessary to provide a complete picture of high-energy processes.

Synchrotron radiation varies inversely as the fourth power of the mass, so the radiation difficulties of e^+-e^- machines can be avoided by the use of "heavy electrons," muons (μ^+-μ^-), and that possibility is the subject of this paper.

The principal liabilities of muons are their short lifetimes and the large initial phase-space area of a muon beam as produced in π decay. The lifetime τ is given by

$$\tau = 2.197 \times 10^{-6} \frac{E_\mu}{m_\mu} \text{ sec} \quad , \tag{1}$$

where E_μ, m_μ are the muon energy and mass. However, this is ~0.02 s for 1-TeV $\mu^\pm$ and is adequate for any linac and for high-energy storage rings and rapid cycling synchrotrons (see below). The large phase-space area of a muon beam can be damped using "ionization cooling"[1] (as described below) to a small value suitable for high-luminosity colliders.

In the following sections, we will describe the principles of muon cooling, discuss cooling scenarios and experiments, and then high-energy collider applications.

Muon Cooling

The basic mechanism of μ cooling is displayed in Fig. 1. The muon beam is passed through a material medium in which it loses energy, principally through

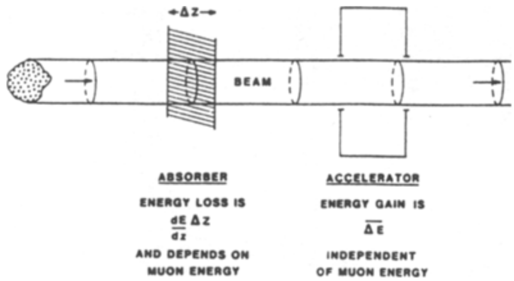

Fig. 1. Sketch of "ionization cooling" principle.

interactions with atomic elections. Following this, it passes through an accelerating cavity where the average longitudinal energy loss is restored. Energy cooling occurs following

$$\frac{d(\Delta E_\mu)}{dn} \cong - \frac{\partial \Delta_\mu}{\partial E_\mu} \Delta E_\mu \quad , \tag{2}$$

where ΔE_μ is the muon energy deviation from the central value, n is the number, of cooling cycles, Δ_μ is the muon energy loss in the absorber, and the derivative is taken at the central value E_μ. Cooling occurs if the derivative is positive. For $E_\mu \lesssim$0.3 GeV, this energy-loss rate derivative is steeply negative for all absorbing materials, but for $E_\mu \gtrsim$0.5 GeV, it is positive with

$$\frac{\partial \Delta_\mu}{\partial E_\mu} \cong 0.2 \frac{\Delta_\mu}{E_\mu} \quad . \tag{3}$$

The precise value has a weak dependence on the absorber material, and the energies $0.5 \lesssim E_\mu \lesssim 2$ GeV are reasonable energies for muon collection. The μ beam is recirculated through many absorber/accelerator cycles either by a return path (cooling ring) or repeated structure (linac) to obtain the desired final distribution.

Transverse damping also occurs because energy loss is parallel to the particle trajectory, whereas energy gain is longitudinal. Expressing this transverse energy loss in terms of rms emittance, one obtains[2]

$$\frac{d\,\epsilon_\perp}{dn} = - \frac{\Delta_\mu}{E_\mu} \epsilon_\perp \tag{4}$$

for both transverse degrees of freedom.

An exchange in cooling rate between the longitudinal (Eq. 3) and a transverse (Eq. 4) dimension can be obtained if a "wedge" absorber in a nonzero Courant-Snyder[3] dispersion region is used[2] (see Fig. 2). Enhanced energy damping with this method implies decreased transverse damping, whereas the sum of ε_x, ε_y, ΔE damping rates is constant:

$$\sum_{x,y,\Delta E} \frac{E_\mu}{E_{cool,i}} \cong 2.2 \text{ is constant} \quad , \qquad (5)$$

where $E_{cool,i}$ is the e^{-1} damping energy in each dimension, $E_{cool} = n_e{-1}\,\Delta_\mu$.

The process is basically similar to radiation damping in $e^\pm$ storage rings,[4] where energy loss in bending sections by synchrotron radiation is recovered in rf cavities. Radiation damping is limited by quantum fluctuations; similarly, muon cooling is limited by statistical fluctuation in muon-atom interactions in the absorber.

The important difference is that muons decay, and cooling must be completed before decay occurs. The muon lifetime (Eq. 1) can be translated to a path length ($\beta_\mu \cong 1$)

$$L_\mu = 6.59 \times 10^2 \frac{E_\mu}{m_\mu} \text{ meters} \quad , \qquad (6)$$

which can be translated into a number of turns of beam storage,

$$N = \frac{L_\mu}{2\pi \bar{R}} = \frac{L_\mu \bar{B}}{2\pi\, B_\rho} = 297\, \bar{B}\text{ (T) turns} \quad , \qquad (7)$$

where $\bar{B}$ is the ring-averaged bending field and B_ρ the magnetic rigidity $[B_\rho(\text{T-m}) \cong 3.3\, E_\mu \text{ (GeV)}]$. N is independent of E_μ.

Muon cooling is limited by heating due to statistical fluctuations in the number and energy exchange in the muon-electron collisions in the absorber. An estimate of this heating in energy cooling can be obtained by noting that the mean energy, exchange is the mean electron ionization energy I[5]

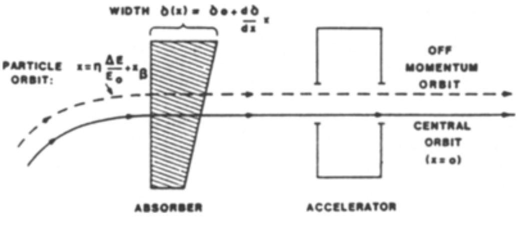

Fig. 2. Use of varying thickness absorber to enhance energy dependence of energy loss.

$$I \cong 10\, Z_{abs} \text{ eV} \quad . \qquad (8)$$

The number of collisions per cooling cycle is $N \sim \Delta_\mu/I$ and the rms energy spread is $\sim\sqrt{N}\, I$ or $\sqrt{\Delta_\mu I}$. Combining cooling with heating, we obtain

$$\frac{d}{dn}\langle(\Delta E)^2\rangle \cong -\frac{2\Delta_\mu}{E_{cool,z}}\langle(\Delta E)^2\rangle + \Delta_\mu I \quad , \qquad (9)$$

which has an equilibrium solution indicating the limits of muon energy cooling,

$$\langle(\Delta E)^2\rangle_{rms} \approx \frac{I\, E_{cool,z}}{2} \quad . \qquad (10)$$

For typical values ($E_{cool,z} = 2$ GeV),

$$\overline{\Delta E_f} = \sqrt{\langle(\Delta E)^2\rangle_{rms}} \cong 10^5 \sqrt{Z_{abs}} \text{ eV} \quad . \qquad (11)$$

Transverse cooling is severely limited by multiple small-angle elastic scattering, mostly Coulomb scattering from the nuclei. The mean scattering angle in passing through an absorber of thickness δ can be estimated by the following equation:[5]

$$\theta_{rms} \cong \frac{14 \text{ (MeV)}}{E_\mu \text{ (MeV)}}\sqrt{\frac{\delta}{L_R}} \quad , \qquad (12)$$

where L_R is the radiation length of the absorber material. The cooling equation for transverse emittance can then be written as

$$\frac{d\varepsilon_x}{dn} = -\frac{\Delta_\mu}{E_{cool,x}}\varepsilon_x + \frac{\beta_x}{2}\left(\frac{14}{E_\mu}\right)^2 \frac{\delta}{L_R} \quad , \qquad (13)$$

where β_x is the C-S betatron function at the absorber. The equilibrium emittance is

$$\varepsilon o = \frac{\beta_x}{2}\left(\frac{14}{E_\mu}\right)^2 \frac{E_{cool,x}}{\left(\frac{dE}{dz} L_R\right)} \quad . \qquad (14)$$

The product $\left(\frac{dE}{dz} L_R\right)$ depends upon the absorber and is largest for light elements[5] (~100 MeV for Be or C but ~7 MeV for Pb or W). The appearance of β_x in Eq. (14) indicates optimum emittance cooling requiring very strong focusing to a low value of β_x at the absorber. Note that the length of the absorber must be less than

~2 β_x, and obtaining maximum absorption in minimum length requires heavy elements, opposing the previous constraint.

We note here that the constraint $L_{abs} \lesssim 2\ \beta_x$ can be relaxed if the absorber is an active focusing element (such as a lithium lens). Optimum μ coolers will probably include such elements in some portion of their structure.

Muon Cooler-Design Outlines and Experiments

In this section we outline some feasible μ cooler designs and experiments. Two basic approaches are suggested: storage ring and linacs. We expect that optimum designs to obtain minimum phase space will combine these in a multistage system.

We first consider a storage-ring system. Figure 3 shows the basic components: a rapid-cycling p synchrotron for π production, a π-decay line, and a storage ring for 1-GeV μ. The muon storage ring is a relatively modest device with conventional magnets (<2T) and modest cooling goals suitable for a μ-p collider.

A second stage would probably be necessary to achieve the lower transverse phase-space densities necessary for a μ^+-μ^- collider; it is difficult to cool transverse emittance by more than a factor of ~30 in a single dc storage ring because of the focusing required in the absorber. A second stage using superconducting magnets ($B \gtrsim 10T$) will obtain $\varepsilon_\perp(1\ \mathrm{GeV}) \lesssim 2.0$ mm-mrad. Higher fields and optimized designs will obtain smaller $\varepsilon_\perp$, but $\varepsilon_\perp \lesssim 0.5$ mm-mrad appears impractical. We note that μ cooling requirements are ideal for use of maximum field superconducting magnets (dc operation, low-particle flux, modest sizes).

Focusing requirements are relaxed in a μ linac where magnet apertures can be reduced, providing stronger focusing, as the beam is cooled. Beam loss from decay is also reduced. Figure 4 shows a modest first-stage μ cooling (100 π → 10 π) linac using conventional magnets. A second stage with superconducting magnets can obtain $\varepsilon_\perp \lesssim 2\ \pi$ mm-mrad.

Existing ~1-GeV storage rings may be modified with low-beta insertions and additional rf for experiments testing μ cooling concepts. Acceleration on the order of 10 MeV/turn is required to obtain cooling before decay. One candidate is the Fermilab 600-MeV/c "electron cooling ring," outfitted with ~5 to 10 MeV of rf borrowed from the future "Debuncher." The $\bar{p}$ production target can be used to provide π's to be transported in a decay line to provide μ's. Another candidate is a SLC damping ring with additional rf and some μ source (instead of e^+).

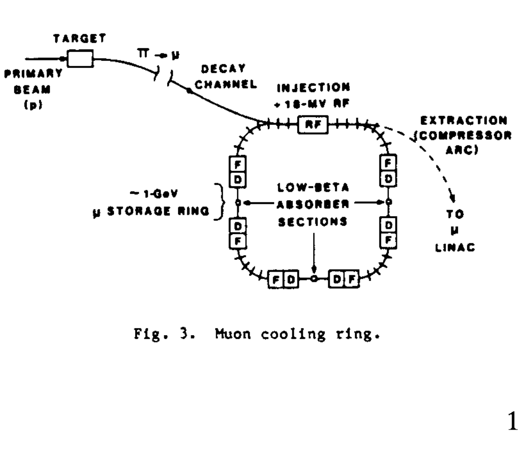

Fig. 3. Muon cooling ring.

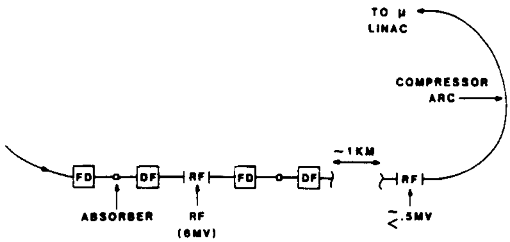

Fig. 4. Sketch of muon cooling linac.

Application of Cooled Muons in High-Energy Accelerators

Cooled muon beams have many possible uses in high-energy accelerators. In this section, we will emphasize applications not accessible to $e^\pm$ machines, such as colliders at $\gtrsim$0.5-TeV energies.

A. μ^+-μ^- Rapid-Cycling Collider

Most collider applications will require a high-intensity muon source at a frequency matched to the muon lifetime. At 1 TeV, $\tau_\mu \cong 0.02$ s, this is reasonably well matched to a high-intensity 30- to 60-Hz rapid-cycling proton synchrotron.

In Fig. 5 we outline the major components of a 1-TeV μ collider: a rapid-cycling proton synchrotron with target to produce π's; a decay channel (or "stochastic injection" into a storage ring)[6] for π → μ decay; a storage-ring/linac system for μ cooling; and a μ linac (or "booster") for injection into a rapid-cycling synchrotron with period matched to the proton synchrotron. In this example the μ synchrotron is simply a conventional larger version of the proton synchrotron. We assume from previous calculations that $\sim 5 \times 10^{-3}$ stored muons are obtained from each primary proton.

The μ^+-μ^- collider luminosity L may be estimated using

$$L \cong \frac{f_0\ n_t\ n_B\ N^+\ N^-}{4\pi\ \beta^*\ \varepsilon^*} \quad . \tag{15}$$

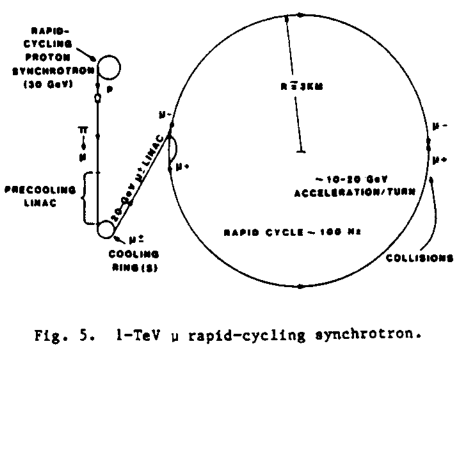

Fig. 5. 1-TeV μ rapid-cycling synchrotron.

With 2×10^{13} protons/pulse, we obtain $\sim 10^{11}$ stored $\mu^{\pm}$ which may be organized into $n_B \cong 2$ bunches with $N^+ = N^- = 5 \times 10^{10}$ μ/bunch. The cycling frequency (30 Hz) is f_o; n_t is the mean number of storage turns (300); and we may estimate $\beta^* \cong 0.3$ cm and $\epsilon^* \approx 2 \times 10^{-7}$ cm-R at 1 TeV. We obtain $L \gtrsim 10^{32}$ cm^{-2} s^{-1}, an adequately high luminosity. The same synchrotron may be used as a μ^--p^+ collider at high luminosity, with more relaxed requirements on N^-, β^*, and ϵ^*. The scenario is, in principle, easier but more expensive at higher energies.

B. Linac/Storage-Ring Scenario

Cooled muons may be suitable for injection in a high-gradient linac. Skrinsky suggested acceleration of μ's in his "proton klystron."[1] Other linac ideas such as "surfatrons," "wake fields," etc., may be more readily adaptable to μ acceleration than e because of the μ immunity to synchrotron radiation, bremsstrahlung and particle-medium interaction.

Assuming a suitable high-gradient linac, μ^+-μ^- (and μ-p) collisions may be obtained in a dc superconducting ring which accepts the high-energy output beam (see Fig. 6). Luminosity can, in principle, be higher than in the previous example ($L \gtrsim 10^{33}$), since stronger fields will increase n_t (the number of beam-storage turns) and decrease β^*, and beam loss in acceleration is reduced.

C. μ-p Colliders

An important advantage of μ^+-μ^- colliders over e^+-e^- is that μ-p collisions may also occur in the same ring. In the rapid-cycling synchrotron protons may be injected with μ^-, and in the storage ring scenario they may be stored before μ^- injection. The revolution frequencies are naturally mismatched because of the different velocities at equal energies. They can be rematched[2] by displacing the beams in energy under the condition (high energy),

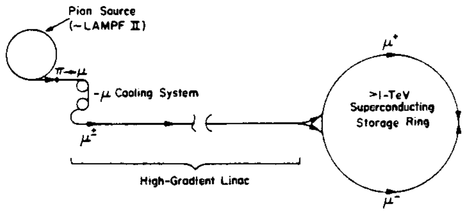

Fig. 6. μ linac/storage-ring system.

$$\frac{\Delta p}{p}\left[\frac{1}{\gamma_t^2}\right] \cong \frac{1}{2\gamma_p^2} - \frac{1}{2\gamma_\mu^2} \quad , \tag{16}$$

where $\Delta p/p$ is the momentum offset, γ_t the transition energy of the ring, and γ_μ, γ_p are the kinetic factors. At 1 TeV with $\Delta p/p = 10^{-3}$ we obtain $\gamma_t \cong 45$, a reasonable value.

References

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