Goal: Measure the emittance of the muon beam to 3% accuracy before and after the muon cooling apparatus.
Possible site: Meson Lab at Fermilab:

Measure 6-D emittance before and after cooling:
Overview

**Measure muons individually**, and form a virtual bunch in software:

⇒ Must know timing to $\approx 10$ psec to select muons properly phased to the 800-MHz RF of the cooling apparatus.

⇒ Use RF accelerating cavity to correlate time with momentum.

⇒ Must measure momentum 4 times.

⇒ Must also have coarse timing ($\lesssim 300$ psec) to remove phase ambiguity.

**Large transverse emittance**, $\epsilon_{N,x} = 1500\pi$ mm-mrad:

⇒ Confine the muon beam in a 3-Tesla solenoid channel.

⇒ All muon detection in the 3-T field.

⇒ Use bent solenoids (toroidal sectors with guiding dipoles) for momentum dispersion.
Muon momentum = 165 MeV/c:

⇒ Larmor period of 1.15 m sets scale for detector arrangement.

⇒ Resolution limited by multiple scattering.

⇒ Perform tracking in a low-pressure gas.

3-T magnetic field ⇒ simplest if detector $E \parallel B$.

⇒ **Time Projection Chambers** (TPC’s)

Higher momentum muons ⇒ higher $B$ and/or larger radius magnets.
- Two TPC’s in same pressure vessel for each of 4 momentum spectrometers.

- Low gas pressure $\Rightarrow$ low operating voltage.

- 1250 cathode pads, 50-MHz timing sampling.

- Analog pipeline via 512-deep switched-capacitor arrays.

- No trigger: capture entire $10 \, \mu$sec window.

- Could process $\approx 10$ tracks $\Rightarrow \approx 1$ MHz rate capability.
Diffusion

Mean free path longer at low pressure ⇒ larger diffusion!

Spatial smearing: \[ \sigma = \sqrt{2Dt} = \sqrt{\frac{2Dz}{v_d}}. \]

But in a magnetic field, transverse diffusion \( \ll \) longitudinal.

Our measurement of \( P_\perp \) in unaffected by longitudinal diffusion.

\[ P = \frac{P_\perp}{\tan \theta} \approx \frac{P_\perp}{\theta} \quad \Rightarrow \quad \frac{\delta P}{P} = \sqrt{\left(\frac{\sigma_{P_\perp}}{P_\perp}\right)^2 + \left(\frac{\sigma_\theta}{\theta}\right)^2}. \]
Longitudinal Diffusion When $E \parallel B$

$$D \parallel \approx \frac{v_d k T}{e E} \quad \text{(Einstein)} \quad \Rightarrow \quad D(E, T) = \frac{T}{T_0} \frac{E_0}{E} D(E_0, T_0).$$

$D \parallel = 10^5 \text{ cm}^2\text{s}^{-1}$ at the saturation velocity $v_d = 10^7 \text{ cm/s}$ in methane at $100^\circ\text{K}$ and 0.01 atmosphere.

$$\Rightarrow \quad \sigma_z = \frac{2D\parallel x}{v_d} = A\sqrt{z}, \quad \text{where} \quad A = 0.135 \text{ cm}^{1/2}.\]$$

Fit for track angle $\theta$ via $u = z/\theta$ where $\hat{u} \perp \hat{z}$ and $U$ is measured on the surface of the helix.

$$\chi^2 = \sum_i \frac{(z_i - u_i/\theta)^2}{\sigma_{z_i}^2} = \sum_i \frac{(z_i - u_i/\theta)^2}{A^2 z_i},$$

$$\Rightarrow \quad \frac{1}{\sigma_{\theta}^2} = \frac{\partial \chi^2}{\partial \theta^2}, \quad \text{and hence} \quad \sigma_{\theta} = A\theta \sqrt{\frac{\theta}{Nz}},$$

$\sigma_{\theta,\text{diffusion}} \approx 0.00006$ for $N = 15$, $z = 45 \text{ cm}$, and $\theta_{\text{rms}} = 0.05$.

$$\Rightarrow \text{Longitudinal diffusion not a problem.}$$
Transverse Diffusion When $E \parallel B$

Field $B \Rightarrow$ transverse mean free path $\lesssim$ Larmor radius.

\[ \Rightarrow \quad D_{\perp} \approx \frac{r_B}{l} D_{\parallel} = \frac{kT}{m\omega_B}. \]

noting \[ \frac{r_B}{l} = \frac{v_d/\omega_B}{v_d\tau} = \frac{1}{\omega_B\tau} \approx \frac{1}{3000}, \quad \text{and} \quad v_d \approx \frac{eE}{m}. \]

\[ \Rightarrow D_{\perp} \approx 33 \text{ cm}^2\text{s}^{-1}, \]

using $\omega_B = 1.8 \times 10^{11} \text{ Hz} \times B \text{ [Tesla]}$, and $B = 3 \text{ T}$.

\[ \Rightarrow \quad \sigma_{\perp, \text{diffusion}}(45 \text{ cm}) \approx \sqrt{\frac{2 \cdot 33 \cdot 45}{10^7}} \text{ cm} = 170 \mu\text{m}. \]

\[ \Rightarrow \text{Transverse diffusion not a problem.} \]
Detector R&D at Princeton

We are now building a small 16-channel low-pressure TPC, which can fit inside an old 6-T magnet that we recently recommissioned.

To study:

1. Accuracy of time and space interpolation via charge sharing on readout pads.


3. Verification of detector performance over long drift paths in a strong magnetic field.

4. Viability of placement of readout electronics next to pad plane (inside the magnetic field).

5. Dynamic range the STAR SCA at 50 MHz (somewhat higher than nominal).