The dynamics of mercury flow in a curved pipe

Yan Zhan
Foluso Ladeinde
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Outline

• Motivation
• Objective
• Previous work
• Scheme of the problem
• Pipe curvature effect
• Laminar flow in the mercury supply pipe
• Conclusion
Motivation

- Liquid **mercury** as a potential high-Z target for Moun Collider Accelerator Project.
- Target delivery systems involves pipe curvature, axially-dependent radius, nozzle diameter and nozzle length etc.
- Proper nozzle design to achieve a less turbulent jet at the nozzle outlet.
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Objective

- Study the dynamics of mercury flow in the target delivery system
- Obtain a basic physical understanding of this internal flow problem for achieving a proper nozzle design
- Start with laminar mercury flow in curved pipe first
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Scheme of the problem
— Equations applied to pipe of arbitrary curvature (1)

\[ \begin{align*}
R &= R(z); \quad C_1 = C_1(z); \quad C_2 = C_2(z) \\
C_1' &= C_1(z) \\
C_2' &= C_2(z)
\end{align*} \]

**Cartesian** \((y_1, y_2, y_3) \xrightarrow{} \text{Curvilinear} \,(z, r, \theta)\)

**Continuity equation**

\[
\frac{R}{R + r \sin \theta} \frac{\partial u_z}{\partial z} + \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\sin \theta}{R + r \sin \theta} u_r + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\cos \theta}{R + r \sin \theta} u_\theta = 0
\]
Scheme of the problem
— Equations applied to pipe of arbitrary curvature (2)

**z-momentum equation**

\[
\frac{\partial u_z}{\partial t} + \frac{R}{R + r \sin \theta} u_z \frac{\partial u_z}{\partial z} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + \frac{\sin \theta}{R + r \sin \theta} u_r u_z + \frac{\cos \theta}{R + r \sin \theta} u_\theta u_z = -\frac{1}{\rho} \frac{R}{R - r \sin \theta} \frac{\partial P}{\partial z}
\]

\[-\nu \left( \frac{1}{r} \frac{\partial}{\partial r} \right) \left( \frac{\partial u_z}{\partial r} + \frac{\sin \theta}{R + r \sin \theta} u_z \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\cos \theta}{R + r \sin \theta} u_z \right) - \left( \frac{1}{r} + \frac{\partial}{\partial r} \right) \frac{R}{R + r \sin \theta} \frac{\partial u_r}{\partial z}
\]

\[-\frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{R}{R + r \sin \theta} \frac{\partial u_z}{\partial z} \right) - \nu \frac{\sin \theta u_r + \cos \theta u_\theta - r \sin \theta (\partial u_z/\partial z)}{(R + r \sin \theta)^3} \frac{R dR}{dz}
\]

**r-momentum equation**

\[
\frac{\partial u_r}{\partial t} + \frac{R}{R + r \sin \theta} u_z \frac{\partial u_r}{\partial z} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \frac{\sin \theta}{R + r \sin \theta} u_z u_r - \frac{1}{\rho} \frac{\partial P}{\partial r}
\]

\[+\nu \left( \frac{1}{r} \frac{\partial}{\partial r} + \frac{\cos \theta}{R + r \sin \theta} \left( \frac{\partial u_r}{\partial r} \right) + \frac{u_\theta}{r} \frac{1}{\partial \theta} \right) - \frac{R^2}{(R + r \sin \theta)^2} \frac{\partial^2 u_r}{\partial z^2}
\]

\[+\frac{R}{R + r \sin \theta} \left( \frac{\partial^2 u_z}{\partial z \partial r} + \frac{\sin \theta}{R + r \sin \theta} \frac{\partial u_z}{\partial z} \right) + \frac{\sin \theta u_z + r \sin \theta (\partial u_r/\partial z)}{(R + r \sin \theta)^3} \frac{R dR}{dz}
\]

**θ-momentum equation**

\[
\frac{\partial u_\theta}{\partial t} + \frac{R}{R + r \sin \theta} u_z \frac{\partial u_\theta}{\partial z} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} - \frac{u_z^2 \cos \theta}{R + r \sin \theta} = -\frac{1}{\rho} \frac{1}{r} \frac{\partial P}{\partial \theta}
\]

\[+\frac{1}{r(R + r \sin \theta)} \frac{\partial^2 u_z}{\partial z \partial \theta} - \frac{R \cos \theta}{(R + r \sin \theta)^2} \frac{\partial u_z}{\partial z} + \left( \frac{\partial}{\partial r} + \frac{\sin \theta}{R + r \sin \theta} \left( \frac{\partial u_r}{\partial r} \right) + \frac{u_\theta}{r} - \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right) \frac{R dR}{dz}
\]

\[+\nu \frac{r \sin \theta (\partial u_\theta/\partial z) + \cos \theta u_z}{(R - r \sin \theta)^3} R dR \frac{dR}{dz}
\]
Scheme of the problem
— Analytic solution for fully developed flow (1)

To get the Analytical solutions, assumptions are needed as follows:

a. Isothermal Newtonian laminar flow
b. Incompressible (does not depend on the pressure)
c. Fully developed \( (d()/dz=0, \text{ except } P ; d()/dt=0) \)
d. Constant small curvature \( (dR/dz=0, a/R << 1) \)
Scheme of the problem
— Analytic solution for fully developed flow (1)

• W.R. Dean’s solution*

\[
\begin{align*}
\frac{u_r}{u_0} &= na \sin \theta (1 - r'^2)^2 (4 - r'^2) / 288 R \\
\frac{u_\theta}{u_0} &= na \cos \theta (1 - r'^2)(4 - 23 r'^2 + 7 r'^4) / 288 R \\
\frac{u_z}{u_0} &= (1 - r'^2)[1 - \frac{3r \sin \theta}{4R} + \frac{n^2 r \sin \theta}{11520 R}(19 - 21 r'^2 + 9 r'^4 - r'^6)]
\end{align*}
\]

Where \( u_0 = Aa^2, n = Aa^3 / \nu, r' = r / a \),

\( A \) is a constant referring to the pressure gradient.

Further the stream function in the pipe cross-section is

\[ \sec \theta = kr'(1 - r'^2)^2 (1 - r'^2 / 4) \]

Where \( k \) is an arbitrary constant.

* W.R. Dean, Note on the motion of fluid in a curved pipe, Imperial College of Science, 1927
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• Pipe curvature effect
  – Straight pipe
  – Curved pipes ($\delta=0.5; \delta=0.013$)
  – Comparisions
• Laminar flow in the mercury supply pipe
• Conclusion
Pipe curvature effect
— Straight pipe (1)

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>1000</th>
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<tbody>
<tr>
<td>Pipe diameter</td>
<td>1.127 mm</td>
</tr>
<tr>
<td>Pipe length</td>
<td>150a</td>
</tr>
<tr>
<td>Inlet condition</td>
<td>Uniform inlet velocity 0.1m/s and static pressure of 18.5bar</td>
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<tr>
<td>Mesh1 ((N_z \times N_r \times N_\theta))</td>
<td>Axial direction 500 (uniform) Radial direction 48 ((\Delta=0.01)) Circumferential direction 24 Total 576000</td>
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<tr>
<td>Mesh2 ((N_z \times N_r \times N_\theta))</td>
<td>Axial direction 1000 (uniform) Radial direction 56 ((\Delta=0.005)) Circumferential direction 24 Total 1344000</td>
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</table>
Pipe curvature effect
— Straight pipe (2)

Fig.1 Mesh comparison for the axial velocity along the center line
Pipe curvature effect
— Straight pipe (3)

Fig. 2 Axial velocity profile comparison for different mesh
Pipe curvature effect
— Curved pipe (1)

<table>
<thead>
<tr>
<th>Reynolds number</th>
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<tbody>
<tr>
<td>Pipe diameter</td>
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<tr>
<td>Pipe Length</td>
<td>20 diameter before bend and 60 diameter after bend</td>
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<tr>
<td>Inlet condition</td>
<td>Fully developed velocity profile and static pressure of 18.5bar</td>
</tr>
</tbody>
</table>

Mesh for Curvature1
(δ1=0.5)
- Axial direction: 586
- Radial direction: 56 (Δ=0.01)
- Circumferential direction: 24
- Total: 787584

Mesh for Curvature2
(δ2=0.013)
- Axial direction: 1560
- Radial direction: 56 (Δ=0.005)
- Circumferential direction: 24
- Total: 2096640
Pipe curvature effect
— Curved pipe (2)

Fig. 3 Numerical results for pipe of curvature of 0.5 at the inlet part
Pipe curvature effect
— Curved pipe (3)

Fig. 4 Numerical results for pipe of curvature of 0.5 at the bend part
Pipe curvature effect
— Curved pipe (4)

Fig. 5 Numerical results for pipe of curvature of 0.5 after the bend part
Pipe curvature effect
— Curved pipe (5)

Fig. 6 Numerical results for pipe of curvature of 0.5 at the outlet part
Pipe curvature effect
— Curved pipe (6)

Fig. 7 Numerical results for pipe of curvature of 0.013 at the inlet part
Pipe curvature effect
— Curved pipe (7)

Fig. 8 Numerical results for pipe of curvature of 0.013 at the bend part
Pipe curvature effect
— Curved pipe (8)

Fig.9 Numerical results for pipe of curvature of 0.013 after the bend part
Pipe curvature effect
— Curved pipe (9)

Fig.10 Numerical results for pipe of curvature of 0.013 at the outlet part
Pipe curvature effect

Comparison

Fig. 11: Axial velocity profile compared at different positions of these two pipes.
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Laminar flow in mercury supply pipe (1)
Laminar flow in mercury supply pipe (2)

<table>
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<tr>
<th>Reynolds number</th>
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<td>Pipe diameter</td>
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<tr>
<td>Curvature radius</td>
<td>1.165a</td>
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<tr>
<td>Inlet condition</td>
<td>Fully developed velocity profile (0.1 m/s) and static pressure of 18.5 bar</td>
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</table>

<table>
<thead>
<tr>
<th>Mesh ( (N_z \times N_r \times N_\theta) )</th>
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<td>Zone C</td>
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<td>Zone D</td>
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<tr>
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<td>Total</td>
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</tbody>
</table>
Laminar flow in mercury supply pipe (3)
Laminar flow in mercury supply pipe (3)
Laminar flow in mercury supply pipe (3)
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Simple conclusions

• Larger curvature pipe affects further upstream and downstream.

• Four vortices show in the large curvature pipe.