Physics Opportunities with Muon Beams: 
Neutrino Factories and Muon Colliders

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http://puhep1.princeton.edu/~mcdonald/mumu/NSFLetter/
Past Uses of Muon Beams:

- Measurement of $g - 2$ of the muon.
- Search for “forbidden” processes: $\mu \rightarrow e\gamma$, $\mu N \rightarrow eN$, ...
- Study of nuclear structure via $\mu N \rightarrow \mu X$.

New Opportunities:

- Neutrino factories based on $\mu \rightarrow e\nu_\mu \bar{\nu}_e$.
  - Neutrino oscillations.
  - Nucleon structure via $\nu_\mu N \rightarrow \mu X$; $X$ includes charm...
  - A path to muon colliders.
- Muon colliders.
  - $s$-channel production of light Higgs.
  - Precision studies of electroweak/supersymmetry physics.
    [Leptonic initial state;
    Beamstrahlung suppressed by $(m_e/m_\mu)^2$.]
  - A new path to the energy frontier.
Oscillations of Massive Neutrinos

Neutrinos could have a small mass (Pauli, Fermi, Majorana, 1930’s).

Massive neutrinos can mix (Pontecorvo, 1957).

In the example of only two massive neutrinos, with mass eigenstates $\nu_1$ and $\nu_2$ with mass difference $\Delta m$ and mixing angle $\theta$, the flavor eigenstates $\nu_a$ and $\nu_b$ are related by

$$
\begin{pmatrix}
\nu_a \\
\nu_b
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}.
$$

The probability that a neutrino of flavor $\nu_a$ and energy $E$ appears as flavor $\nu_b$ after traversing distance $L$ in vacuum is

$$
P(\nu_a \rightarrow \nu_b) = \sin^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 [eV^2]}{E [GeV]} \frac{L [km]}{E [GeV]} \right).
$$

The probability that $\nu_a$ does not disappear is

$$
P(\nu_a \rightarrow \nu_a) = \cos^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 [eV^2]}{E [GeV]} \frac{L [km]}{E [GeV]} \right).
$$
A Sketch of Current Data

• The “anomaly” of atmospheric neutrinos suggests that GeV $\nu_\mu$’s disappear while traversing the Earth’s diameter.
  $\Rightarrow \Delta m^2 \approx 10^{-3} \text{ (eV)}^2$ for $\sin^2 2\theta \approx 1$.  
  (Kamiokande, IMB, Soudan-2, MACRO, Super-Kamiokande)

• The solar neutrino “deficit” suggests that MeV $\nu_e$’s disappear between the center of the Sun and the Earth.
  $\Rightarrow \Delta m^2 \approx 10^{-10} \text{ (eV)}^2$ for $\sin^2 2\theta \approx 1$, if vacuum oscillations.  
  (Homestake, GALLEX, SAGE)

• The LSND experiment at Los Alamos suggests that 30-MeV $\nu_\mu$’s appears as $\nu_e$’s after 30 m.
  $\Rightarrow \Delta m^2 \approx 1 \text{ (eV)}^2$, but reactor data requires $\sin^2 2\theta \lesssim 0.03$.  

The first two results require at least 3 massive neutrinos.

All results together require at least 4 massive neutrinos.

The measured width of the $Z^0$ boson (LEP) $\Rightarrow$ only 3 Standard Model neutrinos. A 4th massive neutrino must be “sterile”.  

The Supersymmetric Seesaw

A provocative conjecture is that neutrino mass $m_\nu$ is coupled to two other mass scales, $m_I$ (intermediate) and $m_H$ (heavy), according to

$$m_\nu = \frac{M_I^2}{M_H}.$$ 

(Gell-Mann, Ramond, Slansky, 1979)

A particularly suggestive variant takes $m_I = \langle \phi_{\text{Higgs}} \rangle = 250$ GeV; Then

$$m_\nu \approx \sqrt{\Delta m^2(\text{atmospheric})} \approx 0.06 \text{ eV} \Rightarrow m_H \approx 5 \times 10^{15} \text{ GeV}.$$ 

This is perhaps the best experimental evidence for a grand unification scale, such as that underlying supersymmetric SO(10) models.

Neutrino oscillations $\Rightarrow$ Supersymmetry.
Mixing of Three Neutrinos

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\]

where \(c_{12} = \cos \theta_{12}\), etc. (Maki, Nakagawa, Sakata, 1962).

Three massive neutrinos ⇒ six independent parameters:

- Three mixing angles: \(\theta_{12}, \theta_{13}, \theta_{23}\),
- A phase \(\delta\) related to CP violation,
- Two differences of the squares of the neutrino masses.

Ex: \(\Delta m_{12}^2 = \Delta m^2(\text{solar})\) and \(\Delta m_{23}^2 = \Delta m^2(\text{atmospheric})\).

Measurement of these parameters is a primary goal of experimental neutrino physics.

If four massive neutrinos, then 6 mixing angles, 3 phases, 3 independent squares of mass differences.

[Theorists find the MNS matrix more analyzable than the CKM matrix.]
Matter Effects

$\nu_e$’s can interact with electrons via both $W$ and $Z^0$ exchanges, but other neutrinos can only interact via $Z^0$ exchange.

$$\Rightarrow \sin^2 2\theta_{\text{matter}} = \frac{\sin^2 2\theta_{\text{vac}}}{\sin^2 2\theta_{\text{vac}} + (\cos 2\theta_{\text{vac}} - A)^2},$$

where $A = 2\sqrt{2}G_F N_e E / \Delta m^2$ depends on sign of $\Delta m^2$.

At the “resonance”, $\cos 2\theta_{\text{vac}} = A$, $\sin^2 2\theta_{\text{matter}} = 1$ even if $\sin^2 2\theta_{\text{vac}}$ is small (Wolfenstein, 1978, Mikheyev, Smirnov, 1986).

$\Rightarrow$ 3 MSW solutions to the solar neutrino problem:
Too Many Solutions

There are 8 scenarios suggested by present data:

• Either 3 or 4 massive neutrinos.

• Four solutions to the solar neutrino problem:

  1. Vacuum oscillation (VO) solution;
     \[ \Delta m_{12}^2 \approx (0.5 - 5.0) \times 10^{-10} \text{ eV}^2, \sin^2 2\theta_{12} \approx (0.7 - 1.0). \]

  2. Low (Just So) MSW solution;
     \[ \Delta m_{12}^2 \approx (0.5 - 2.0) \times 10^{-7} \text{ eV}^2, \sin^2 2\theta_{12} \approx (0.9 - 1.0). \]

  3. Small mixing angle (SMA) MSW solution;
     \[ \Delta m_{12}^2 \approx (4.0 - 9.0) \times 10^{-6} \text{ eV}^2, \sin^2 2\theta_{12} \approx (0.001 - 0.01). \]

  4. Large mixing angle (LMA) MSW solution;
     \[ \Delta m_{12}^2 \approx (0.2 - 2.0) \times 10^{-4} \text{ eV}^2, \sin^2 \theta_{12} \approx (0.65 - 0.96). \]

• Atmospheric neutrino data ⇒ \[ \Delta m_{23}^2 \approx (3 - 5) \times 10^{-4} \text{ eV}^2; \]
  \[ \sin^2 \theta_{12} > 0.8. \]

• \[ \theta_{13} \] very poorly known; \[ \delta \] completely unknown.
The Next Generation of Neutrino Experiments

- Short baseline accelerator experiments (miniBoone, ORLAND, CERN) will likely clarify the LSND result.

- Super-Kamiokande + new long baseline accelerator experiments (K2K, Minos, CERN) will firm up measurements of $\theta_{23}$ and $\Delta m^2_{23}$, but will provide little information on $\theta_{13}$ and $\delta$.

- New solar neutrino experiments (BOREXino, SNO, HELLAZ, HERON, ....) will explore different portions of the energy spectrum, and clarify possible pathlength-dependent effects. SNO should provide independent confirmation of neutrino oscillations via comparison of reactions $\nu^+\overline{2H} \rightarrow p + p + e$ and $\nu^+\overline{2H} \rightarrow p + n + \nu$.

- Each of these experiments studies oscillations of only a single pair of neutrinos.

- The continued search for the neutrinoless double-beta decay $^{78}\text{Ge} \rightarrow^{78}\text{Se} + 2e^-$ will improve the mass limits on Majorana neutrinos to perhaps as low as 0.001 eV (hep-ex/9907040).
The Opportunity for a Neutrino Factory

- Many of the neutrino oscillation solutions permit study of the couplings between 2, 3, and 4 neutrinos in accelerator based experiments.

- More neutrinos are needed!

- Present neutrino beams come from $\pi, K \rightarrow \mu\nu_\mu$ with small admixtures of $\nu_\mu$ and $\nu_e$ from $\mu$ and $K \rightarrow 3\pi$ decays.

- Higher (per proton beam power), and better characterized, neutrino fluxes are obtained from $\mu$ decay.

Collect low-energy $\mu$’s from $\pi$ decay, accelerate the $\mu$’s to the desired energy, and store in a ring while they decay via $\mu^- \rightarrow e^-\nu_\mu\bar{\nu}_e$. [Of course, can use $\mu^+$ also.]
6 Classes of Experiments at a Neutrino Factory

\[ \nu_\mu \rightarrow \nu_e \rightarrow e^- \quad \text{(appearance)}, \quad (1) \]
\[ \nu_\mu \rightarrow \nu_\mu \rightarrow \mu^- \quad \text{(disappearance)}, \quad (2) \]
\[ \nu_\mu \rightarrow \nu_\tau \rightarrow \tau^- \quad \text{(appearance)}, \quad (3) \]
\[ \nu_e \rightarrow \nu_e \rightarrow e^+ \quad \text{(disappearance)}, \quad (4) \]
\[ \nu_e \rightarrow \bar{\nu}_\mu \rightarrow \mu^+ \quad \text{(appearance)}, \quad (5) \]
\[ \nu_e \rightarrow \bar{\nu}_\tau \rightarrow \tau^+ \quad \text{(appearance)}. \quad (6) \]

[Plus 6 corresponding processes for \( \bar{\nu}_\mu \) from \( \mu^+ \) decay.]

Processes (2) and (5) are easiest to detect, via the final state \( \mu \).

Process (5) is noteworthy for having a “wrong-sign” \( \mu \).

Processes (3) and (6) with a final state \( \tau \) require \( \mu \)’s of 10’s of GeV.

Processes (1) and (4) with a final state electron are difficult to detect.

Finely segmented, magnetic detectors of 10’s of kilotons will be required.
The Rates are High at a Neutrino Factory

<table>
<thead>
<tr>
<th></th>
<th>( \nu_\mu )</th>
<th>( \nu_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Neutrino Factory</strong></td>
<td>(2 ( \times ) 10^{20} ( \nu_\mu )/yr)</td>
<td></td>
</tr>
<tr>
<td>10 GeV</td>
<td>2200</td>
<td>1300</td>
</tr>
<tr>
<td>20 GeV</td>
<td>18,000</td>
<td>11,000</td>
</tr>
<tr>
<td>50 GeV</td>
<td>( 2.9 \times 10^5 )</td>
<td>( 1.8 \times 10^5 )</td>
</tr>
<tr>
<td>250 GeV</td>
<td>( 3.6 \times 10^7 )</td>
<td>( 2.3 \times 10^7 )</td>
</tr>
</tbody>
</table>

**MINOS (WBB)**

<table>
<thead>
<tr>
<th></th>
<th>( \nu_\mu )</th>
<th>( \nu_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low energy</td>
<td>460</td>
<td>1.3</td>
</tr>
<tr>
<td>Medium energy</td>
<td>1440</td>
<td>0.9</td>
</tr>
<tr>
<td>High energy</td>
<td>3200</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Even a low-energy neutrino factory has high rates of electron neutrino interactions.

A neutrino factory with \( E_\mu \gtrsim 20 \text{ GeV} \) is competitive for muon neutrino interactions.
Scaling Laws for Rates at a Neutrino Factory

\[ \sigma_\nu \propto E; \quad I_\nu \propto 1/(\theta)^2 \propto (E/L)^2: \quad \text{Rate} \propto I_\nu \sigma_\nu \propto E^3/L^2. \]

⇒ Rate \( \propto E^3 \) at fixed \( L \), \quad \text{Rate} \propto 1/L^2 \) at fixed \( E \).

Neutrino oscillation probability varies with \( L/E \),

⇒ Rate \( \propto E \) for fixed \( L/E \).

\[ \Delta m_{23}^2 = 3 \times 10^{-3} \text{ eV}^2 \]
\[ \sin^2 2\theta_{23} = 1 \]

\( \tau \) appearance suppressed at low energy. Larger \( E \) ⇒ larger \( L \).
\[ \nu_\mu \rightarrow \nu_\mu \rightarrow \mu^- \] **Disappearance**

\[ E_\mu = 30 \text{ GeV}, \]
\[ 2 \times 10^{20} \mu \text{ decays}, \]
\[ L = 7000 \text{ km}, \]
\[ \sin^2 2\theta_{23} = 1. \]

(hep-ph/9906487)

\[ \Delta m_{23}^2 \quad \text{Events} \]
\[ \quad (\text{eV}^2) \quad (\text{per 10 kt-yr}) \]
\[ 0.002 \quad 2800 \]
\[ 0.003 \quad 1200 \]
\[ 0.004 \quad 900 \]
\[ 0.005 \quad 1700 \]
\[ \text{No Osc.} \quad 6200 \]

\[ \nu_\mu \rightarrow \nu_\tau \rightarrow \tau^- \] **Appearance**

\[ \Delta m_{23}^2 \quad \text{Events} \]
\[ \quad (\text{eV}^2) \quad (\text{per 10 kt-yr}) \]
\[ 0.002 \quad 1200 \]
\[ 0.003 \quad 1900 \]
\[ 0.004 \quad 2000 \]
\[ 0.005 \quad 1800 \]

For conditions as above.
Measuring $\theta_{13}$

Many ways:

\[ P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \frac{1.27\Delta m_{23}^2L}{E_\nu}, \]

\[ P(\nu_e \rightarrow \nu_\tau) = \sin^2 2\theta_{13} \cos^2 \theta_{23} \sin^2 \frac{1.27\Delta m_{23}^2L}{E_\nu}, \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = \cos^4 \theta_{13} \sin^2 2\theta_{23} \sin^2 \frac{1.27\Delta m_{23}^2L}{E_\nu}. \]

10 kton detector,
$E_\mu = 20$ GeV,
$2 \times 10^{20}$ $\mu$ decays,
$L = 732$ km,
$\sin^2 2\theta_{23} = 1$,
Left: $\nu_e \rightarrow \nu_\mu \rightarrow \mu^+$,
Right: $\nu_\mu \rightarrow \nu_\mu \rightarrow \mu^-$,
Box = presently allowed. (hep-ph/9811390).
Measuring the Sign of $\Delta m_{23}^2$ via Matter Effects

The matter effect resonance depends on the sign of $\Delta m^2$ (p. 7).

Large effect of $\Delta m_{23}^2$ in $\nu_\mu$ (disappearance) if $\sin^2 2\theta_{13} \approx 0.1$.

For smaller $\sin^2 2\theta_{13}$, may be better to use $\nu_e \rightarrow \nu_\mu$ (appearance).
Measuring $\delta$ via CP Violation

The phase $\delta$ is accessible to terrestrial experiment in the large mixing angle (LMA) solution to the solar neutrino problem (or if there are 4 massive neutrinos).

CP violation:

$$A_{\text{CP}} = \frac{P(\nu_e \rightarrow \nu_\mu) - P(\nu_e \rightarrow \bar{\nu}_\mu)}{P(\nu_e \rightarrow \nu_\mu) + P(\bar{\nu}_e \rightarrow \nu_\mu)} \approx \frac{2\sin \delta}{\sin 2\theta_{13}} \sin \frac{1.27\Delta m^2_{12}L}{E},$$

assuming $\sin^2 2\theta_{12} \approx \sin^2 2\theta_{23} \approx 1$ (LMA).

10 kton detector,

$2 \times 10^{21}$ muon decays,

Large angle MSW:

$\Delta m^2_{12} = 10^{-4}$ eV$^2$,

$\Delta m^2_{23} = 2.8 \times 10^{-3}$ eV$^2$,

$\theta_{12} = 22.5^\circ$,

$\theta_{13} = 13^\circ$,

$\theta_{23} = 45^\circ$,

$\delta = -90^\circ$.

(hep-ph/9909254)

Matter effects dominate the asymmetry for $L > 1000$ km.
Measuring $\delta$ via T Violation

If the small mixing angle (SMA) solutions holds, may still be able to measure $\delta$ via T violation:

$$P(\nu_e \rightarrow \nu_\mu) - P(\nu_\mu \rightarrow \nu_e) =$$

$$4J \left( \sin \frac{1.27 \Delta m^2_{12} L}{E} + \sin \frac{1.27 \Delta m^2_{13} L}{E} + \sin \frac{1.27 \Delta m^2_{23} L}{E} \right),$$

$$J = \frac{1}{8} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta = \text{Jarlskog invariant.}$$

Matter effects could make $\sin 2\theta_{12}$ resonance for $E \approx 100$ MeV and $L \approx 10,000$ km (hep-ph/9911258).

However, not easy to measure $\nu_\mu \rightarrow \nu_e \rightarrow e^- \text{ (appearance)}$ against background of $\nu_e \rightarrow \nu_e \rightarrow e^+$ in a large, massive detector in which the electrons shower immediately. [Rates low also.]
Controlling the $\nu_e$ Flux via Muon Polarization

For $\mu^-$ decay in flight,

$$
\frac{dN_{\nu\mu}(\theta_{\nu\mu} = 0)}{dx} = 2Nx^2[(3 - 2x) + P(1 - 2x)], \\
\frac{dN_{\nu e}(\theta_{\nu e} = 0)}{dx} = 12Nx^2(1 - x)(1 + P),
$$

where $x = 2E_\nu/m_\mu$, and $P$ is the muon polarization.

$[\theta_\nu = 0 \Rightarrow \text{colinear decay; at } P = -1, \text{ all colinear decays forbidden for } \theta_{\nu e} = 0, \text{ but one is allowed for } \theta_{\nu \mu} = 0.]$

Modulate the muon polarization to modulate the relative rates of $\nu_\mu \rightarrow \nu_e \rightarrow e^-$ and $\nu_e \rightarrow \nu_e \rightarrow e^+$. 

(Blondel, http://alephwww.cern.ch/~bdl/muon/nufacpol.ps)
Summary

• The physics program of a neutrino factory/muon collider is extremely diverse, and of scope to justify an international laboratory.

• The first step is a neutrino factory capable of systematic exploration of neutrino oscillations.
  – With $\gtrsim 10^{20} \, \nu$’s/year can go well beyond other existing or planned accelerator experiments.
  – Beams with $E_{\nu_e} \lesssim 1$ GeV are already very interesting.
  – Higher energy is favored: Rate $\propto E$ at fixed $L/E$; $\nu_\tau$ appearance practical only for $E \gtrsim 30$ GeV.
  – Detectors at multiple distances needed for broad coverage of parameter space $\Rightarrow$ triangle or “bowtie” storage rings.
  – CP and T violation accessible with $\gtrsim 10^{21} \, \nu$’s/year.
  – Control of muon polarization extremely useful when studying $\nu_e \rightarrow e$ modes.