Thermal Hall Effect from Neutral Currents in Quantum Magnets

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1. Berry curvature and finite Hall effect in charge-neutral currents
2. The frustrated pyrochlore magnet Tba2Ti2O7
3. Kagome ferromagnet Cu(1,3 -- bdc)

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Predictions:
Chiral spin texture leads to $K_{xy}$ in sparse lattices (bonds between asymmetric plaquettes) --- Kagome lattice and pyrochlores

Heuristic derivation of $K_{xy}$ in spin liquid (with fermionic spinons)

Forgot magnetization current?
Onose et al. observed a weak $K_{xy}$ in insulating pyrochlore ferromagnet $\text{Lu}_2\text{V}_2\text{O}_7$
Luttinger’s gravitational potential approach to $K_{ij}$

**Problem of calculating $K_{ij}$ in linear response (Kubo) theory**

$(-\nabla T$ is a statistical force, not dynamical)

Luttinger (1964) considered photons in a gravitational potential $\Psi(r)$. Blue-shifted photons transport an energy current $J_E$. At equilibrium, $J_E = -J_Q$.

Instead of $J_Q$, we calculate $J_E$ by adding $\Psi(r)$ to $H$ in Kubo approach.

Just add to $H$ the term ($h(r)$ is energy density)

$$\int d\mathbf{r} \; h(\mathbf{r}) \; \Psi(\mathbf{r})$$

$$H = H_0 + V$$
$$V = -e\mathbf{E}.\mathbf{r} + \frac{1}{2}(\mathbf{Hr} + \mathbf{rH}).\nabla\Psi/c^2$$

In $\mathbf{B}$ field, this leads to a large $K_{xy}$ correction from magnetization current (Streda ’77, ’85)
Berry Curvature

Consequence of ignoring inter-band transitions by constraining dynamics to lowest band.

Karplus Luttinger (1954) discovered the term $\mathbf{A}(\mathbf{k})$ in theory of AHE

Wannier coord.

$$ \mathbf{x} = \mathbf{R} + \mathbf{A}(\mathbf{k}) $$

Adams-Blount coordinate (1960)

$A(\mathbf{k}) = (u_k, i\nabla u_k)$ (Berry vector potential)

$$ [x_i, x_j] = i\epsilon^{ijk} \Omega_k $$

$\Omega(\mathbf{k}) = \nabla \times A(\mathbf{k})$ (Berry curvature)

anomalous velocity $v_A$

Berry curvature $\Omega(\mathbf{k})$ acts like a magnetic field in $\mathbf{k}$-space (vanishes if both time reversal and inversion symm hold)

$\Omega(\mathbf{k})$ is a key concept in Topolog. Insuls., Weyl and Dirac semimetals, valley physics, etc
Theoretical Prediction of a Rotating Magnon Wave Packet in Ferromagnets

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\[ \dot{r}_n = \frac{1}{\hbar} \frac{\partial \varepsilon_{nk}}{\partial k} - \hat{k} \times \Omega_n(k), \quad \hbar \dot{\hat{k}} = -\nabla U(r), \]

Usual Kubo term

Magnetization term

\[ L_{ij}^{xy} = -\frac{(k_B T)^2}{\hbar V} \sum_{n,k} \Omega_{n,z}(k) c_d^{\dagger}(\rho_n), \]

(a) Magnon wavepacket

(b) Edge current of magnon

(c) Edge current of magnon

(d) Thermal Hall current of magnon

\[ \nabla T \]
Experiments on a frustrated quantum pyrochlore magnet (spin liquid candidate)
And an ordered Kagome magnet

1) Pyrochlore $\text{Tb}_2\text{Ti}_2\text{O}_7$
   High-temp suscep yields Curie-Weiss MFT temp of 19 K but
   fails to order down to 50 mK.
   Ground state “Quantum spin ice”, may harbor spin liquid
   Investig. for 20 years (mostly neutron), but still an enigma

2) Kagome magnet $\text{Cu}(1,3 – bdc)$
   Kagome planes of Cu separated by organic molecule
   Orders magnetically at 1.8 K
Pyrochlores, spin-ice systems

Two-in, two-out config.

Classical Spin ice
Dy$_2$Ti$_2$O$_7$, Ho$_2$Ti$_2$O$_7$

Quantum spin ice (no trace of 2-in/2-out)
Tb$_2$Ti$_2$O$_7$, Yb$_2$Ti$_2$O$_7$

I) Thermal conductivity vs temp (B=0) in Tb$_2$Ti$_2$O$_7$

A very poor thermal conductor below 5 K
Magneto-thermal conductance $T < 20$ K

Large contribution to thermal current from spin excitations

Dominant below $\sim 3$ K

Very large field effect

Metamagnetic transition at $H_s = 2$ T leads to step-like increase in $K_{xx}$
Hall effect in a neutral current?? Experimental checks

**Checks**

- Hall signal reverses when gradient ($-\nabla T$) is reversed in same $B$
- Hall signal scales linearly with gradient strength
- Hall signal is 1000 x larger than in nonmag analog $Y_2Ti_2O_7$
Thermal Hall conductivity $10 < T < 140 \text{ K}$
Thermal Hall conductivity $1 < T < 15$ K
Below 3 K, $\tan \theta_H$ is $H$-linear (with constant slope) until $H > H_p$.

Defines low-$T$ state with largest Hall response.

The state is readily destroyed when $H$ exceeds $H_p(T)$. 

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**Hall angle**

![Graph](image)
Phase diagram of large Hall state in the H-T plane

Extent of large-Hall response state in T-H plane (shaded).

Hall response is strongly suppressed in field-induced metamagnetic state ($H>H_s$)
The Quantum Spin Ice

The Quantum Excitations in Quantum Spin Ice

Coulomb ferromagnet (spinons deconfined)
Quantum spin liquid (spinons deconfined)

Higgs phases (spinons condensed)

Fractional excitations in CFM and QSL states
E field monopoles
B field monopoles
Faux photon

Hermele, Fisher, Balents (PRB 2004)
Rare example of Kagome magnet

Cu(1,3-\textit{bdc}), benzenedicarboxylate or \textit{bdc}

Becomes antiferromagnetic (type I) below 1.8 K
K vs. Temp in Kagome Magnet (H=0)

Max Hirschberger, Robin Chisnell, Young Lee and NPO
K vs. Temp in Kagome Magnet

A

\( \kappa_{\text{W/Km}} \)

T (K)

Sample 1
Sample 2
Sample 3

B

\( \kappa_{\text{W/Km}} \)

\( \kappa/T \)

\( \kappa T_{\text{W}/\text{K}^2\text{m}} \)

T (K)

Sample 3
Rich behavior of $K_{xx}$ vs $H$ at low temp ($T < 4$ K)
Thermal Hall conductivity in Kagome magnet
$K_{xy}$ reverses sign as a function of $T$, and as a function of $H$.

Rules out skew scattering of phonons
Berry curvature and Chern number of adjacent magnon bands

Lee, Han and Lee, preprint
Computed Kxy of Kagome Magnet

Calculation captures main qualitative features, some quantitative discrepancies
$K_{xy}$ and near-Minkowski gravity, neutral modes

K$_{xy}$ may probe neutral fractional excitations

1) FQHE,  
Kane and Fisher (1996), Read and Green (2001)

2) Neutral Topological Insulators,  
Ryu, Moore, Ludwig (2012); Stone (2013)  
Nomura, Ryu, Furusaki, Nagaosa (2012)

Near-Minkowski

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

Lorentz force

\[ \nabla \times \mathbf{B}_g = -4\pi G J_m/c^2 \]
gravitomagnetic field

Frame dragging

\[ K_{xy} = \left(\frac{\pi^2}{6}\right) k_B^2 T / 2h \]

Ryu, Moore, Ludwig  
Nomura, Ryu, Furusaki, Nagaosa  
M. Stone
Summary

Observation of a large Hall effect from neutral spin excitations

I) In frustrated magnet Tb$_2$Ti$_2$O$_7$
   Excitations are not magnons
   Are they spinons? Other fractional excitations?

II) In Kagome ferromagnet Cu(1-3, bdc)
   Hall signal observed both above and below Tc
   Unexpected sign reversal at Tc
   Consequence of Berry curvature in different magnon bands

III) Other spin liquid and exotic quantum magnets?
   Related pyrochlore Yb$_2$Ti$_2$O$_7$
   Herbert smithite
   ET salts
   *dmit* salts
   Han Purple salt SrCu(BO$_3$), Haldane Shastry systems
   Classical spin ice Dy$_2$Ti$_2$O$_7$
Experimental checks