

# Frequency-Dependent Characteristics of the EME Path

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The fundamental nature of radio-frequency echoes from the moon was well established half a century ago. Professional studies were made at Jodrell Bank, MIT Lincoln Laboratory, Arecibo Observatory, and elsewhere, motivated by a desire to evaluate potential applications of EME (Earth-moon-Earth) communication — possibly even broadcasting! — and for surreptitious eavesdropping on the private VHF/UHF radio traffic of others. These ideas soon gave way to more practical solutions using man made satellites. The applications of lunar radar evolved into astronomical studies of terrain and surface composition, both for scientific reasons and to evaluate possible landing sites for the Apollo astronauts.

Much of this early work is known to the amateur EME community, at least in broad outline, but some is not. In this paper I summarize the basic professional results of most interest to radio amateurs, with special emphasis on the effects of lunar libration. I then present a number of new measurements, all made with amateur EME equipment, to show that libration effects on EME signals are neither mysterious nor unpredictable. Indeed, the statistical effects of frequency spreading and multipath fading — two different ways of describing the same libration-induced phenomenon — can be predicted accurately for any time, frequency, and locations on Earth. These results show clearly why 15-to-20 WPM CW signals propagated by EME are often hard to copy on the 432 and 1296 MHz bands, because of libration fading, even when signal strength is otherwise adequate. The results provide useful quantitative constraints for making optimum choices of coding and modulation for the EME path.

## Fundamentals of Lunar Radar

A typical lunar radar experiment of the early 1960's transmitted high-power pulses a few microseconds wide, and examined the time structure of their return echoes. The relevant geometry is illustrated in Figure 1. As suggested by the sketch of returned echo power, the full duration of lunar echoes is  $t = 2R/c = 11.6$  ms: that is, twice the moon's radius,  $R = 1738$  km, divided by  $c$ , the speed of light. However, most of the received power comes from near the leading edge of the echo, from a bright central spot of radius  $r \ll R$ . Thus, in back-scattered radio waves the lunar disk is bright near its center and dark at the edge, the lunar "limb" — a very different situation from that for visible light, where there is very little limb darkening and the full moon is almost equally bright over the full disk. The reason, of course, is that on distance scales comparable to the relevant radio wavelengths — a few centimeters to ten meters or so — most of the lunar surface is relatively smooth. In contrast, on scales of visible wavelengths the surface is rough and scatters light efficiently in all directions.

The geometric quantities illustrated in Figure 1 are related by the following equations:

$$r = R \sin \varphi$$
$$d = R \left[ 1 - \left( 1 - \frac{r^2}{R^2} \right)^{\frac{1}{2}} \right] = R (1 - \cos \varphi)$$
$$t = \frac{2d}{c}$$

These equations give specific one-to-one relationships between lunar radius  $R$ , angle of incidence  $\varphi$ , projected distance  $r$  from the center of the disk, range depth  $d$ , and excess time delay  $t$ .

Hagfors (*Radio Science*, vol. 5, pp. 189–227, 1970) published detailed data on lunar reflectivity (or radar “cross section”) per unit area of the disk. By integrating those data from the center of the disk outward, one obtains the curves of Figure 2. It’s easy to see that at 430 MHz, half of the available echo power comes from the innermost 14% of the disk radius, or 2% of disk area. Small but measurable amounts of power come from farther out, toward the lunar limb. By contrast, at 1.3 GHz the half-power point extends to about 19% of the full disk radius, and at 8 GHz to 33%. In terms of time delay, half of the power is received in the first 120  $\mu$ s at 430 MHz, the first 210  $\mu$ s at 1.3 GHz, and the first 650  $\mu$ s at 8 GHz.

## Lunar Librations

Most amateur EME stations are not configured for transmitting narrow pulses, or for the fast T/R switching needed to make direct measurements of echo power with millisecond resolution. However, another measurement approach is possible, related to lunar librations. It is well known that the moon’s spin is synchronized to its orbital motion, so that approximately the same face is presented to Earth at all times. The lunar orbit is eccentric, so the orbital angular speed varies according to Kepler’s law. On the other hand, the spin rate is constant, so the moon’s orientation appears to “wobble” back and forth in a monthly cycle. The orbit is inclined to the ecliptic (the Earth’s orbital plane) by about  $5.1^\circ$ , which introduces a monthly North-South “nodding” component of motion.

An even larger contribution to apparent lunar wobble is caused by Earth rotation. This motion has a daily cycle, and its amplitude is proportional to the cosine of observer latitude. The net effect of all these motions gives the moon an apparent direction and rate of spin at any specified time and viewing location on Earth. An important consequence, as indicated in Figure 3, is that reflecting regions on one side of the lunar disk may be approaching the observer, while those on the other side are receding. These motions cause differential Doppler shifts from reflecting regions spread across the disk, and thus frequency spreading of returned echoes. The frequency spread causes signal fading as individual contributions interfere constructively or destructively. For angular libration rate  $\omega$ , the rotational velocity at the moon’s equator is  $R\omega$ . Thus, for a transmitted signal at frequency  $f$  the maximum Doppler offset is  $f_0 = 2fR\omega/c$  and the full limb-to-limb Doppler spread is  $2f_0 = 4fR\omega/c$ . Since we know that most of the echo power comes from a relatively small fraction of the disk, we should expect the frequency profile of an EME echo to be sharply peaked toward its center but to have broad, low-level wings.

The libration rate  $\omega$  can be calculated for any time and any location on Earth by techniques similar to those used to find the moon’s position in the sky. The bottom panel of Figure 4 shows such calculations for the K1JT location (Maidenhead locator FN20qi) over the month of August, 2010. The two upper panels show expanded views of the data for August 7 and August 19-20. The assumed frequency is 1 GHz, and limb-to-limb spreads for other frequencies can be obtained by multiplying by the actual frequency in GHz. Data are plotted only for times when the moon is above the horizon. The curves have a distinct shape for each day, determined by the combination of effects with daily and monthly periods. The detailed shapes will be different for different terrestrial locations.

For a station at  $40^\circ$  latitude the computed Doppler spreads at 1 GHz extend upward to nearly 30 Hz. Deep minima like the one on August 7 (see Figure 4, especially the top panel) occur when the vector sum of all libration contributions approaches zero. These deep minima are precisely the reason for the occasional times when normally rapid QSB slows almost to a stop.

The effect of libration on EME signals between two different stations is the sum of one-way contributions for each station alone. Because  $\omega$  has both magnitude and direction, the summation must be done using vectors before final reduction to a magnitude. Examples of such calculations for three different EME paths are presented in Figure 5. Stations closer to the equator, such as WP4G and PY2SRB, will generally see larger libration rates and thus larger Doppler spreads.

### Amateur Libration Measurements

Can these libration effects be observed and accurately measured with amateur equipment? Indeed they can! Figure 6 presents spectra obtained at 432 and 1296 MHz at the K2UYH station on June 6, 2010, using the program *WSJT8* in its echo-testing mode. The upper two panels have linear power scales, while the lower panels emphasize low-amplitude wings of the spectra by using a scale in dB relative to the maximum. Vertical ticks near the bottom of each panel delimit the computed range of Doppler spreads at  $\pm f_0$ . The measured spectral extents correspond very closely to these limits, thereby confirming that significant echo power was being received, even from close to the lunar limb. As expected, however, the spectral profiles are sharply peaked, showing that most echo power comes from near the center of the disk where Doppler shifts are smallest.

For practical purposes related to amateur EME communication, I define two parameters for measuring frequency spread:  $w_{50}$  and  $w_{10}$ . Both are readily determined from autocorrelation functions of the echo spectra. Loosely speaking, they represent the full widths of those functions at the 50% (-3 dB) and 10% (-10 dB) points. In other words,  $w_{50}$  represents a range of frequencies containing half of the echo power, while  $w_{10}$  contains 90% of the power. For the data shown in Figure 6, the autocorrelation functions yield  $w_{50} = 2.3 \pm 0.3$  Hz and  $w_{10} = 6.9 \pm 0.6$  Hz at 432 MHz, and  $w_{50} = 6.2 \pm 0.3$  Hz and  $w_{10} = 18.9 \pm 0.6$  Hz at 1296 MHz. Many other measurements of  $w_{50}$  and  $w_{10}$  have been made over the past two months, covering wide ranges of frequency, libration rate, and location on Earth. Six more examples are shown in Figure 7, based on data obtained at 144 MHz by K1JT, at 1296 MHz by VK7MO, and at 10.3 GHz by G3WDG. Again, the full limb-to-limb range of predicted Doppler spread is shown for each spectrum by means of short vertical ticks.

Figure 8 assembles the presently available measurements of  $w_{50}$  and  $w_{10}$  for 144 MHz through 2.3 GHz and plots them against limb-to-limb Doppler spread calculated for the time, frequency, and place of observation. The sets of measurements for both  $w_{50}$  and  $w_{10}$  are well represented by straight lines. There is a slight trend toward larger widths at frequencies above 2 GHz. As expected, this effect becomes significantly greater at even higher frequencies.

In passing, I note that the echo measurements reported at 144 and 432 MHz and 10 GHz used linear polarization, while those at 1296 and 2303 MHz used circular (transmitting right-hand and receiving left). Theory says that the depolarized component of return echoes should exhibit larger values of  $w_{50}$  and  $w_{10}$  than the polarized component, because depolarization will be greater at large angles of incidence. It would be interesting to test this prediction, but I have not yet done so.

### Consequences for EME

What does all this mean for the practicalities of amateur EME communication? Figure 9 summarizes the frequency dependence of Doppler spread, the most essential quantity distinguishing real EME signals from those arising from a hypothetical smooth, stationary reflector. The sloping solid line illustrates the maximum limb-to-limb Doppler spread for self-echoes from a station at latitude  $\pm 40^\circ$ . Typical spreads will be smaller than this maximum, and sometimes much smaller; the long-dashed straight line therefore gives median values of limb-to-limb Doppler spread. (Half the time, the effect

will be greater than the median; half the time it will be less.) Finally, the short-dash and dash-dot curves illustrate expected median values of  $w_{50}$  and  $w_{10}$ . These curves have been calculated to include the extra frequency dependence arising from larger active reflecting regions at the higher frequencies. Remember that the Earth-rotation contribution to libration rate depends on the observer's latitude. Doppler spreads for stations near the equator will be somewhat larger than those shown, and for high-latitude stations will be less. For two-station exchanges of signals, the locations of both stations are important.

The measurable widths  $w_{50}$  and  $w_{10}$  characterize the range of Doppler spreads of greatest practical consequence for making EME QSOs with weak signals. At 1296 MHz and below, we experience libration effects primarily as signal fading. Most of the fading takes place at rates between  $w_{50}$  and  $w_{10}$ , or timescales between  $1/w_{10}$  and  $1/w_{50}$ . At 2.3 GHz and above most libration fading is faster than Morse code elements, so we don't notice it as such. Instead, we hear the effect as broadening of the signal in the frequency domain. CW EME signals sound increasingly rough as frequency increases above 2 GHz.

The transmission rates of CW dashes and full characters at 20 WPM are illustrated by the horizontal dotted lines in Figure 9. Together with the curves for  $w_{50}$  and  $w_{10}$ , these lines define a problematic region where Morse-coded CW can be notoriously hard to copy. Inside this region, libration-induced fading takes place at about the same rate as the transmission of amplitude-modulated code elements. At lower frequencies, e.g. 144 MHz, fading is slow enough that several CW characters can be heard and copied on signal peaks. Above 2 GHz, fading is mostly faster than the important code elements and we simply hear a "less than T9" CW tone.

Of course, digital protocols such as JT65 are also affected by libration-induced Doppler spreads, but for a number of reasons the consequences can be much less severe than with on-off CW keying. JT65 transmissions have a robust synchronizing mechanism, a known structure, and more than five-fold redundancy built into their Reed-Solomon error-correction scheme. Their modulation is 65-tone frequency-shift keying, with exactly one tone always present. Signal dropouts within a transmission (no tone present) are easily recognized and ignored by the decoder. There is no effect comparable to a Morse dash being effectively chopped into two dits by a brief null. There is no need to piece together copied fragments of a repeated message, since this is done automatically when the decoder identifies a Reed Solomon codeword.

Doppler spread sets a lower limit on the tone spacings usable for frequency-shift keying. The tone spacings for sub-modes JT65A, B, and C are respectively 2.7, 5.4, and 10.8 Hz. With this in mind, Figure 9 shows clearly why JT65B is perfectly usable at 144 and 432 MHz, while JT65C is preferable under most conditions at 1.3 and 2.3 GHz. At 3.4 GHz even JT65C will show degraded performance, except when the libration rate is low. At still higher frequencies, JT65C is useful only at selected times of minimal libration. The mode JT4G, with 315-Hz spacing between each of its four tones, should be effective up to 10 GHz.

## Conclusions

For each of the most popular amateur EME bands the following table lists maximum and median limb-to-limb Doppler spreads for stations at  $\pm 40^\circ$  latitude, followed by measurable widths  $w_{10}$  and  $w_{50}$  that correspond to the median. These numbers can serve as a quick-reference guide to the appropriate magnitude ranges for frequency spreading on each band.

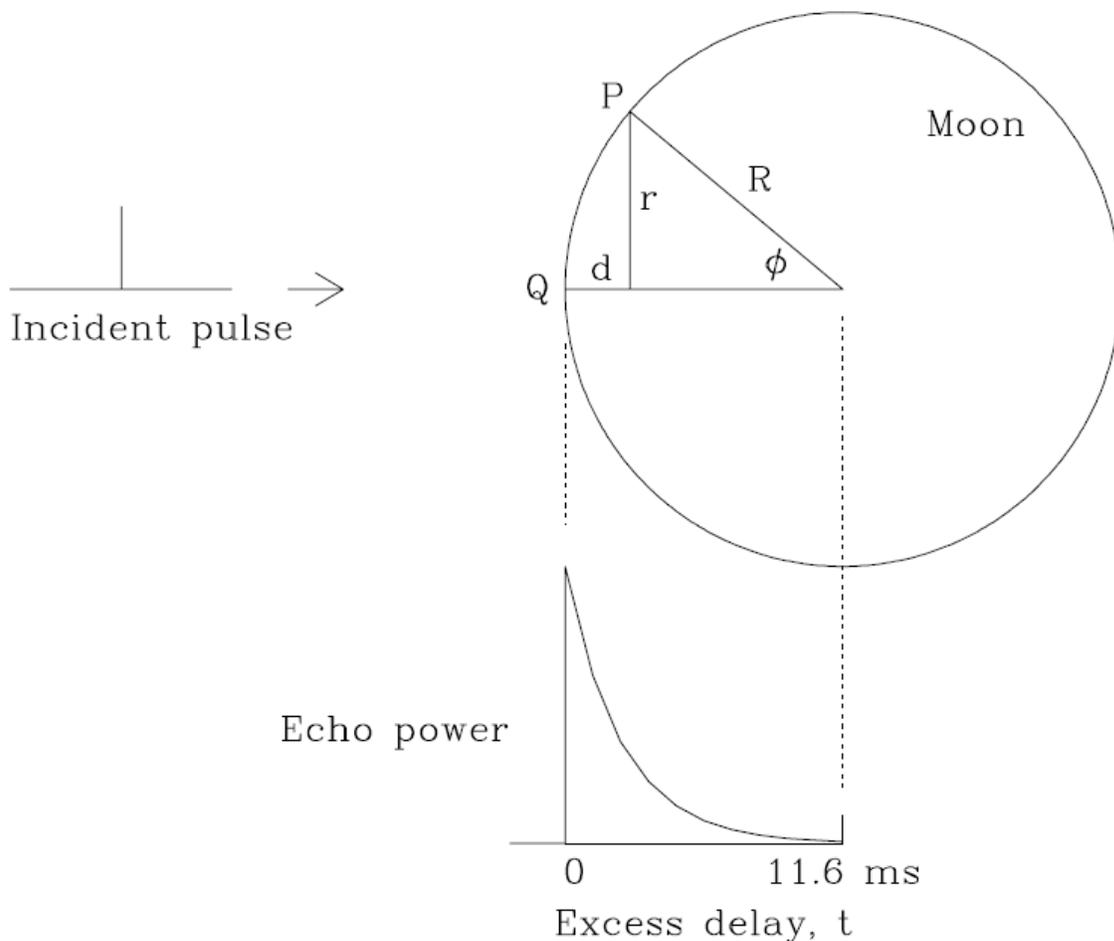
Frequency (MHz)	Limb-to-Limb Maximum (Hz)	Limb-to-Limb Median (Hz)	$w_{10}$ Median (Hz)	$w_{50}$ Median (Hz)
144	4	2.2	1.1	0.4
432	12	7	3	1.2
1296	37	19	10	4
2304	65	35	20	8
3400	97	51	31	12
5760	164	86	60	25
10368	295	156	128	58

Finally, here are some general lessons and rules of thumb relating to the discussion in this paper:

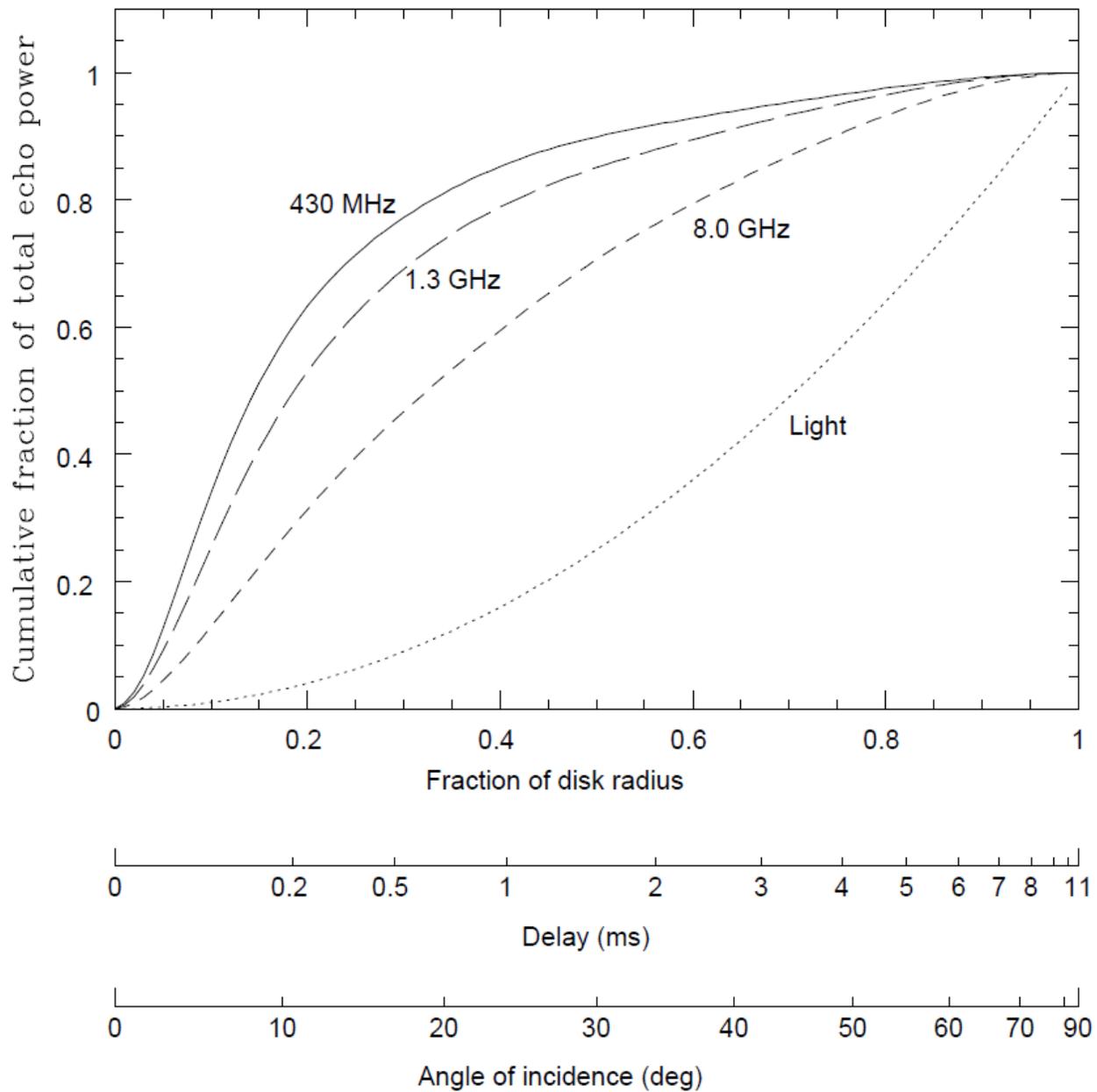
- A smooth moon would produce a specular reflection. Most echo power would come from a few Fresnel zones around disk center. Time spread would be much less than  $2R/c = 11.6$  ms and frequency spread much less than  $30f$  Hz, with  $f$  in GHz. Polarization would be preserved, but the sense of circular polarization would be reversed.
- A rough moon would produce diffuse echoes, with uniform brightness across the disk. Time spread would be 11.6 ms, and limb-to-limb frequency spread as much as  $30f$  Hz, depending on libration rate. There would be significant depolarization, with cross-polarized return echoes just a few dB weaker than the “correct” polarization.
- At the frequencies used for amateur EME, conditions fall somewhere between these two extremes. Lunar echoes are nearly specular at VHF and are increasingly diffuse at higher frequencies.
- Lunar libration is an apparent effect relative to a particular observer. It has an East-West component from the moon’s orbital eccentricity and a North-South component from orbital inclination, both with a monthly period, and a daily East-West contribution from Earth rotation. The net libration rate can be computed for any time and location.
- Libration fading and libration-induced Doppler spread are the same thing, experienced in different ways. Fading time scale is the inverse of Doppler width.
- The inevitable fading of EME signals, especially troublesome at 432 and 1296 MHz, can be mitigated if the signal has sufficient diversity in time, frequency, or both. CW operators use a single narrow-band signal and much repetition, which is better than nothing. Digital modes can use robust synchronization, multiple frequencies, and mathematically encoded redundancies, which together can be much more effective.

## Acknowledgments

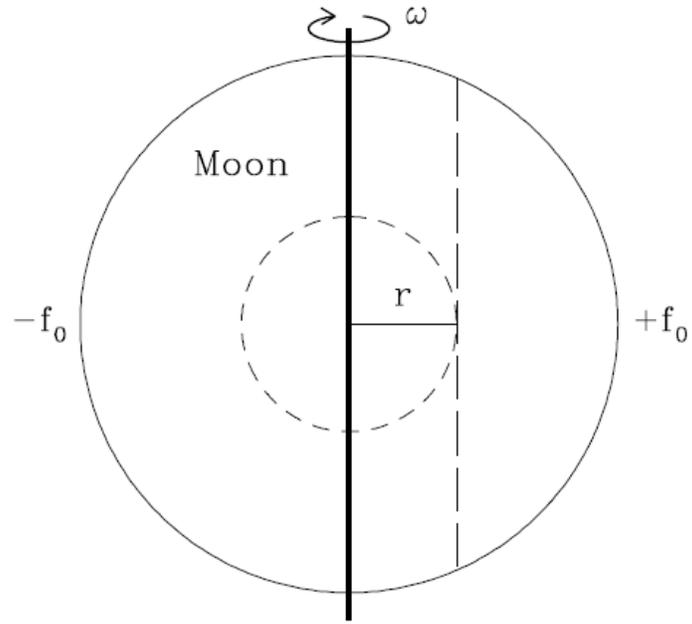
I thank Charlie Suckling, G3WDG, who has drawn recent attention to the existence of predictable libration minima. Charlie also shared some raw data from his 10 GHz echo tests, which I used in Figure 7. David Anderson, GM4JJJ, quickly incorporated the calculation of limb-to-limb Doppler spreads into his fine EME program, *MoonSked*, and I was happy to check my own calculations against those in *MoonSked*. Rex Moncur, VK7MO, used my program *WSJT8* to make echo tests at 1296 MHz and 2.3 GHz, providing data used in Figures 7 and 8. Alan Katz, K2UYH, joined me in making *WSJT8* measurements of frequency spread at 432 and 1296 MHz, at his station. All of this work was carried out in the finest spirit of amateur cooperation, and the help is much appreciated!



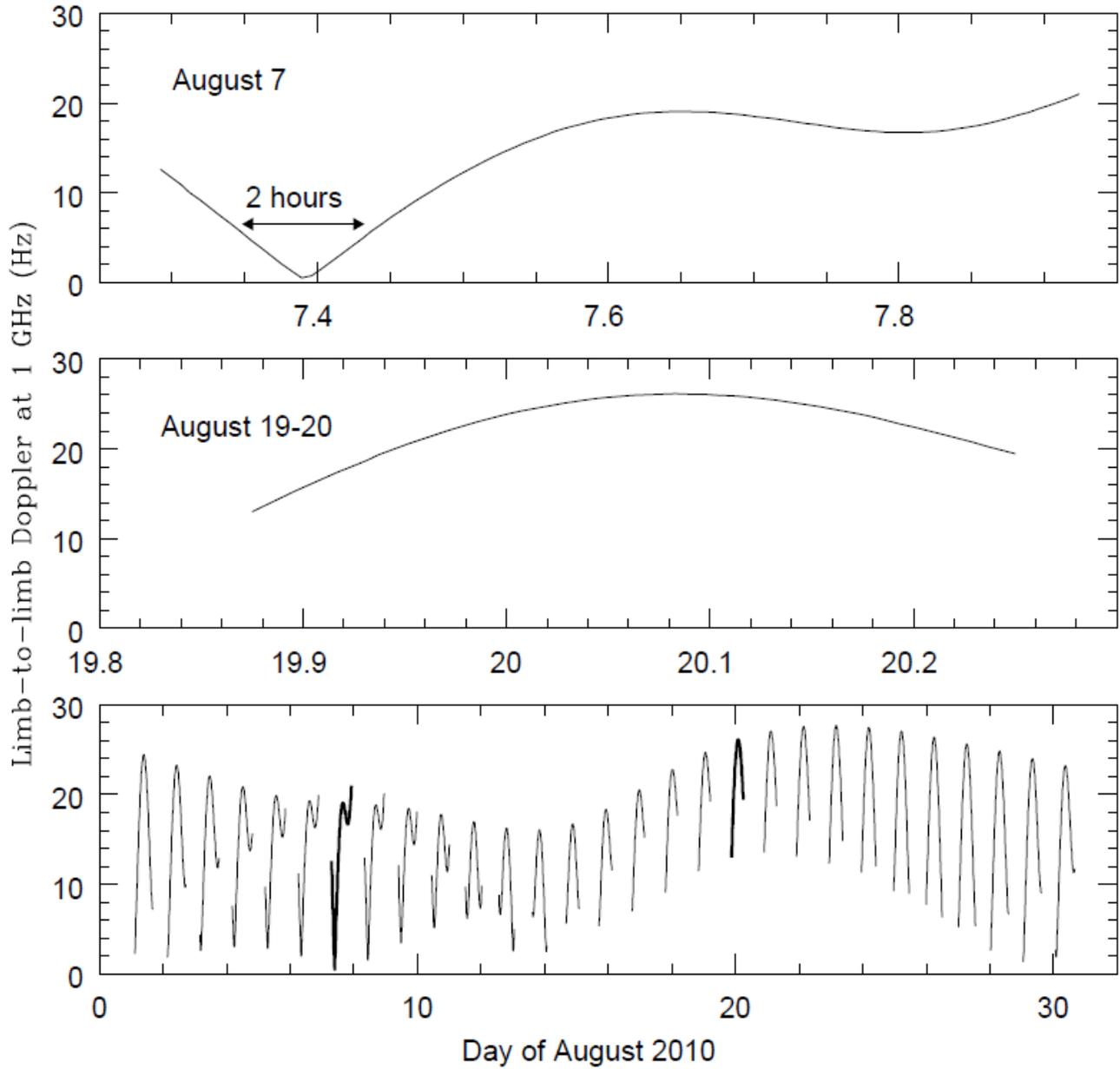
**Figure 1.** Simplified EME geometry, side view. Echo contributions from point P are delayed by  $t = 2d/c$  relative to those from the disk center, point Q. The maximum excess delay is  $2R/c = 11.6$  ms.



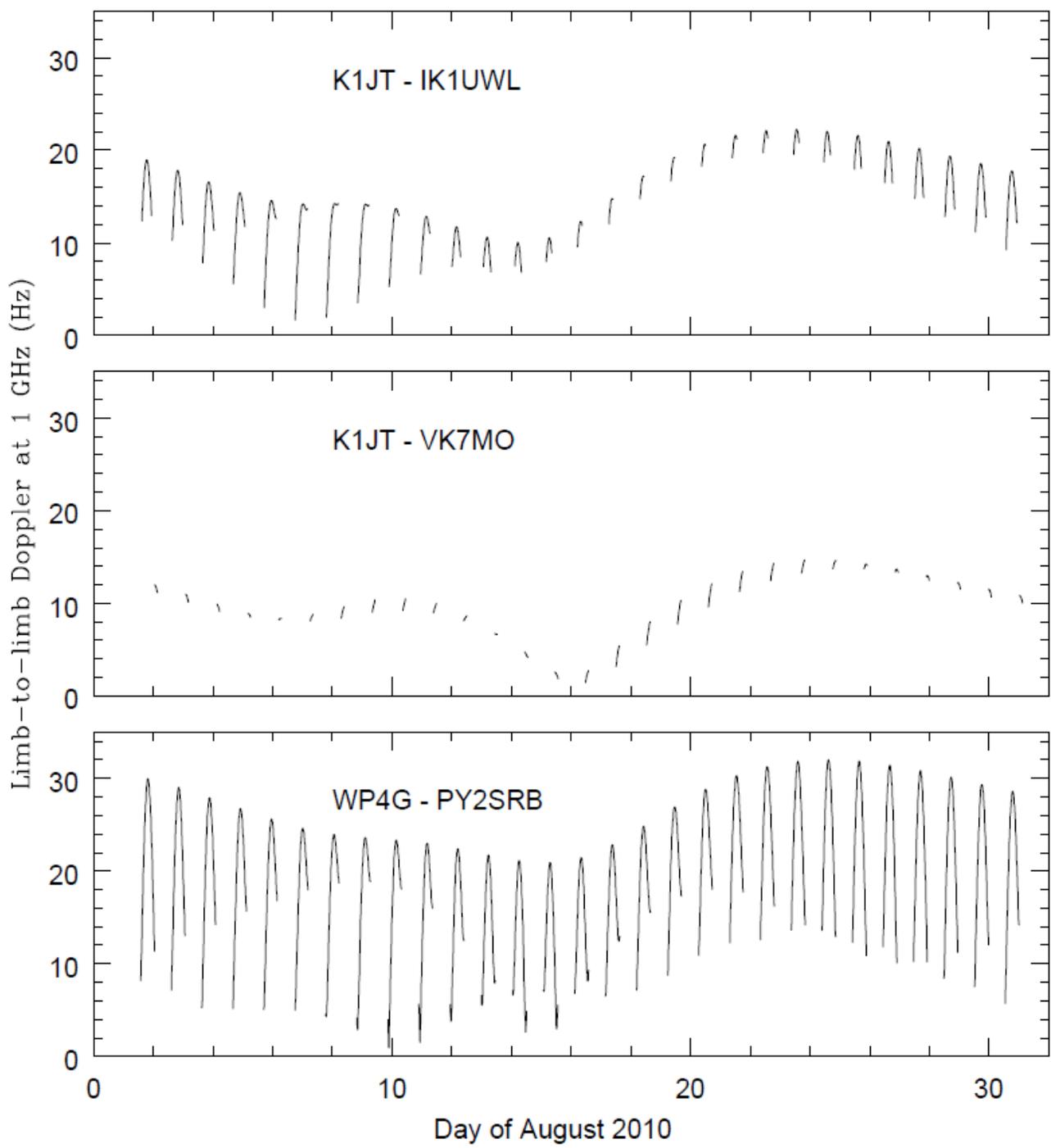
**Figure 2.** Cumulative fraction of echo power received from inside specified fractions of the lunar radius. Corresponding angles of incidence for incoming radiation and excess time delays are shown at the bottom. The radio-frequency data are derived from Hagfors (1970).



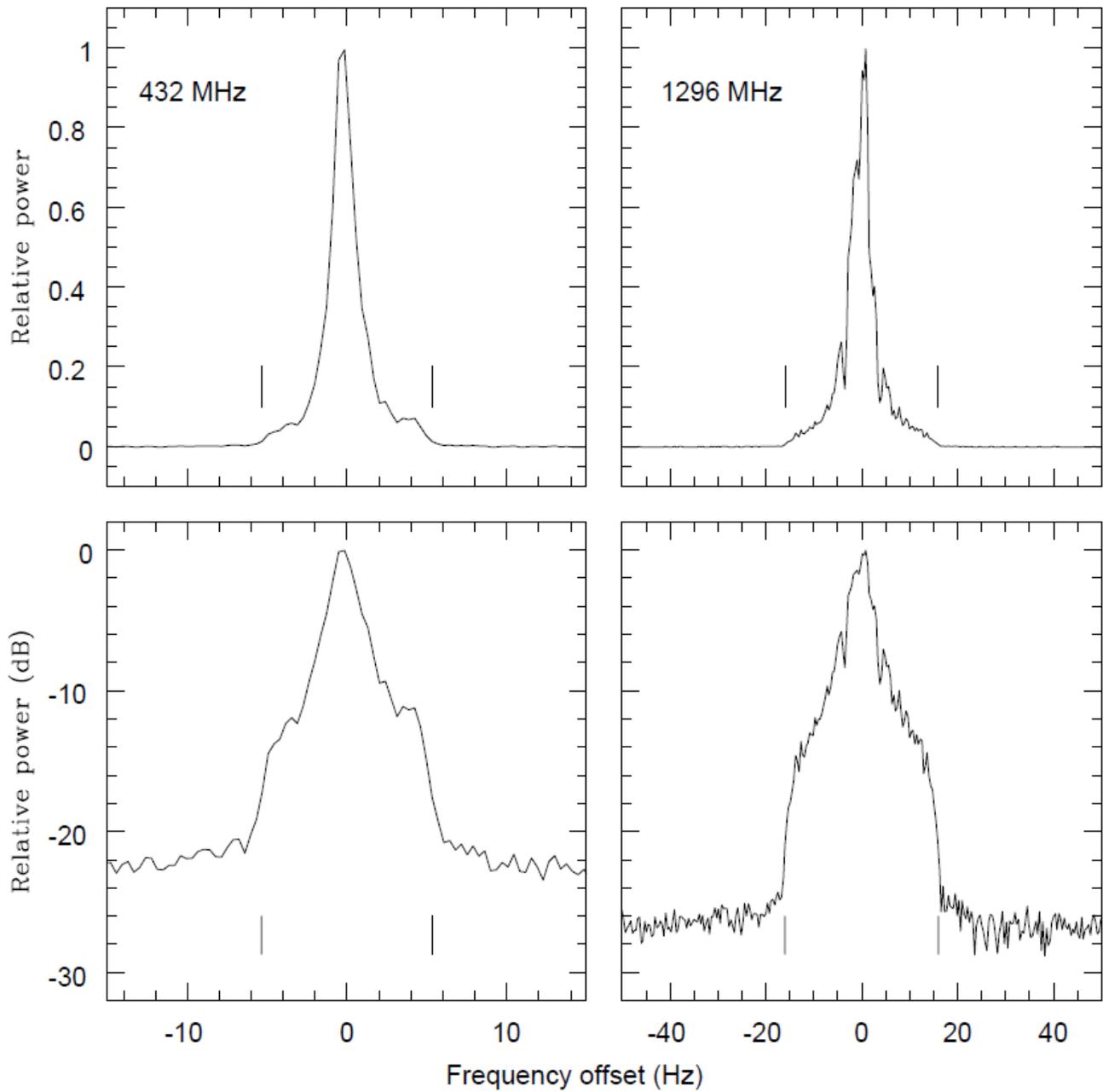
**Figure 3.** EME geometry, viewed from Earth. The ring of radius  $r$  is a locus of constant echo delay; the dashed vertical line is a locus of constant Doppler shift. Maximum positive and negative Doppler shifts occur at the equatorial points marked  $\pm f_0$ .



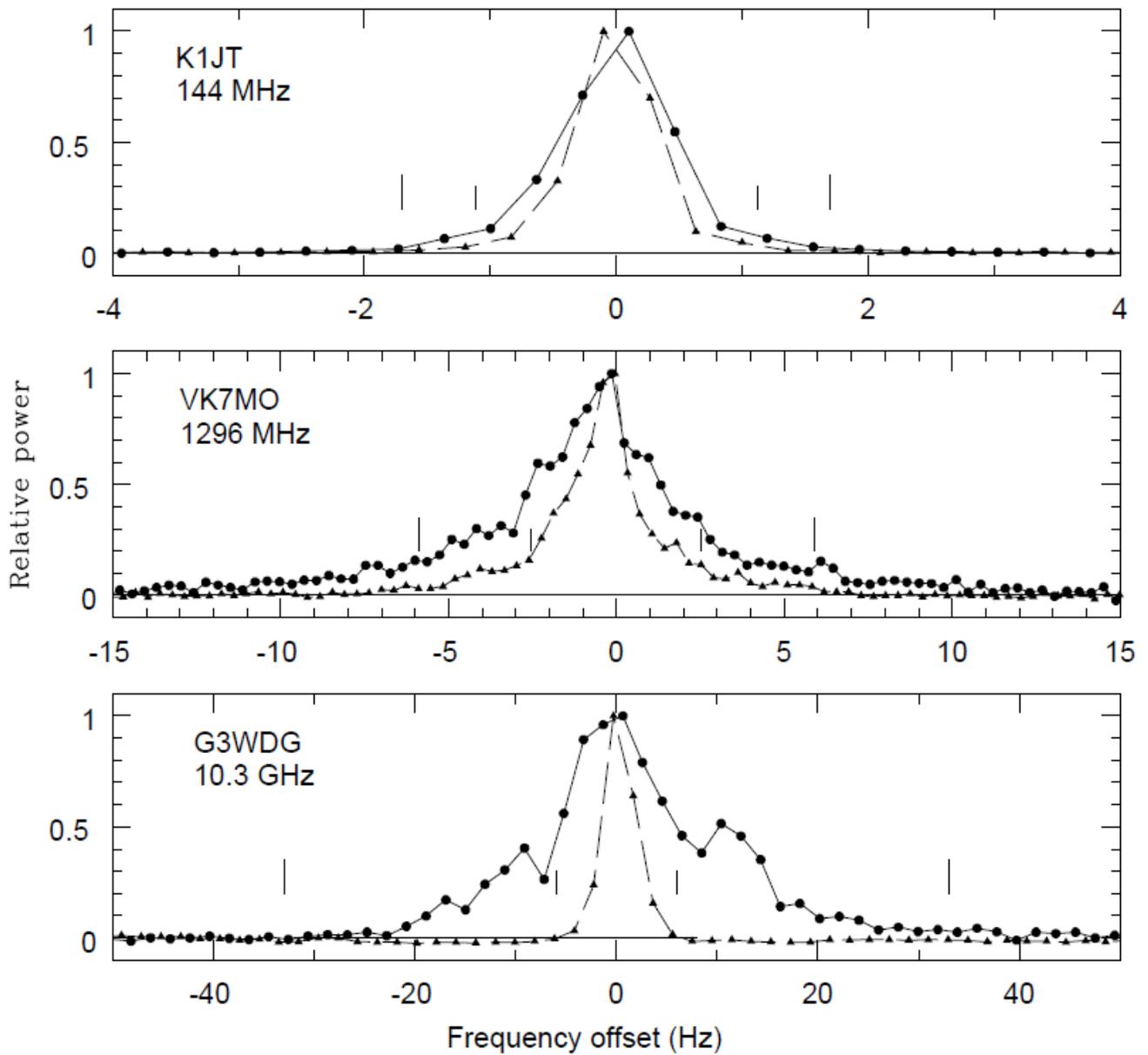
**Figure 4.** Limb-to-limb Doppler shifts calculated for self-echoes at K1JT, for the month of August 2010. Curves for each day extend from moon rise to moon set. The two upper panels give expanded views for August 7 and August 19-20, and curves for these dates are highlighted in the bottom panel. Doppler shifts may be scaled from 1 GHz to other frequencies in the usual way.



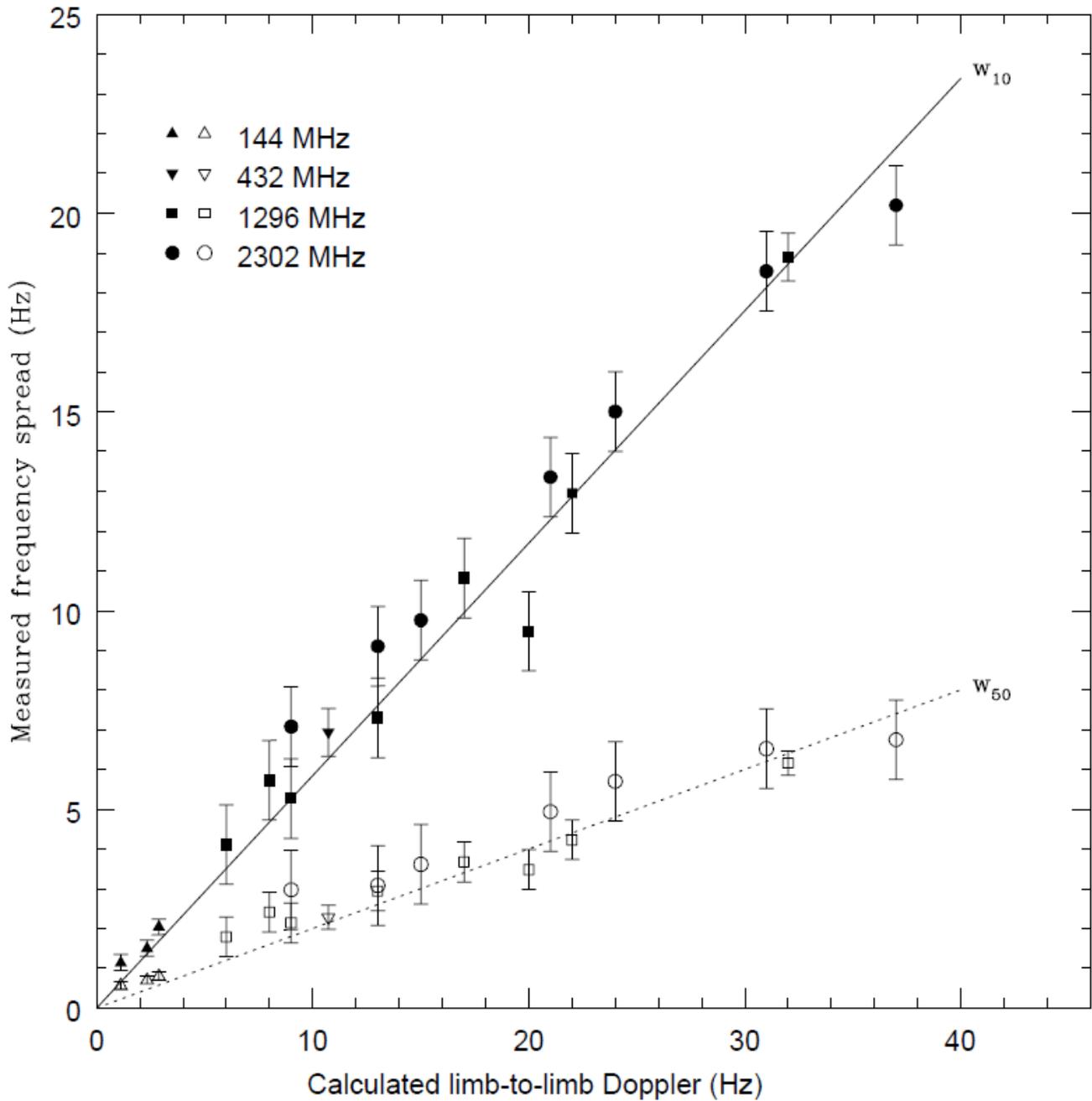
**Figure 5.** Limb-to-limb Doppler shifts calculated for the EME paths K1JT to IK1UWL, K1JT to VK7MO, and WP4G to PY2SRB for the month of August 2010. Curves for each day cover the full range of mutual moon availability.



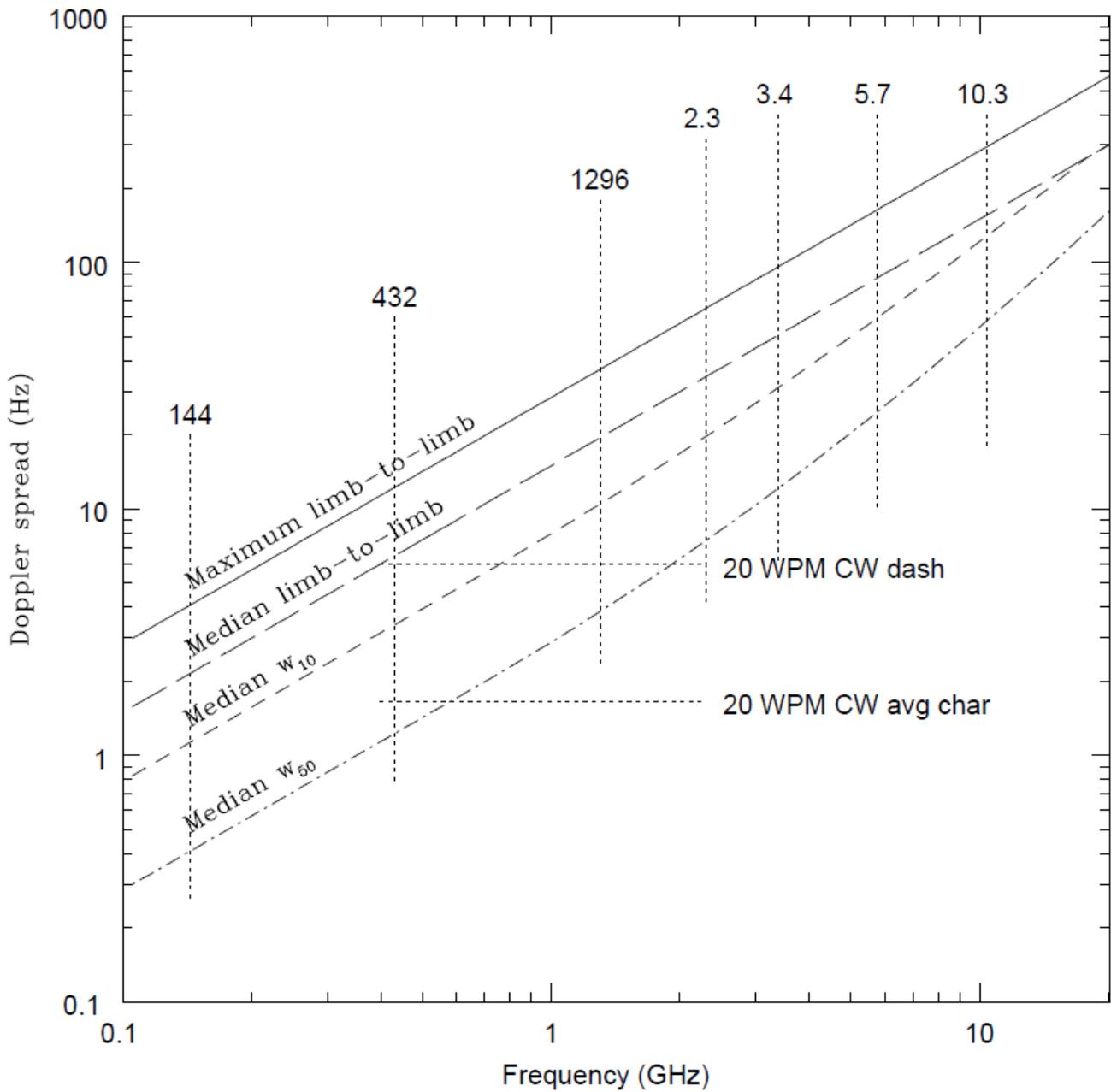
**Figure 6.** Measurements of Doppler spread at 432 and 1296 MHz, from echo tests made at K2UYH on June 6, 2010. The upper panels use a linear power scale; the lower ones are in dB relative to the maximum. Vertical ticks indicate the predicted maximum positive and negative Doppler offsets,  $\pm f_0$ .



**Figure 7.** Measurements of Doppler spread at two different libration rates, at each of three frequencies. The measurements at K1JT were on 2010 May 21 at 19:30 and 23:30 UTC; those at VK7MO on May 26 at 06:30 and 10:30 UTC; and those at G3WDG on May 18 at 08:50 and 09:56 UTC. Vertical ticks indicate the predicted maximum positive and negative Doppler offsets,  $\pm f_0$ , for each measurement.



**Figure 8.** Measured values of frequency spread  $w_{10}$  and  $w_{50}$  at three different frequencies and many different libration rates. The echo tests at 144 MHz were made at K1JT; those at 432 MHz, at K2UYH; those at 1296 MHz at VK7MO and K2UYH; and those at 2302 MHz, at VK7MO.



**Figure 9.** Maximum and median Doppler spreads for self-echoes at latitude  $\pm 40^\circ$ , and the corresponding median values of  $w_{10}$  and  $w_{50}$ . Horizontal dotted lines represent the transmission rates of dashes and individual characters of Morse code sent at 20 words per minute.