“Crossed-Field” and “EH” Antennas Including Radiation from the Feed Lines and Reflection from the Earth’s Surface

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1 Problem

An ongoing challenge in electrical engineering is the design of antennas whose size is small compared to the broadcast wavelength λ. One difficulty is that the radiation resistance of a small antenna is small compared to that of the typical transmission lines that feed the antenna,[1] so that much of the power in the feed line is reflected off the antenna rather than radiated unless a matching network is used at the antenna terminals (with a large inductance for a small dipole antenna and a large capacitance for a small loop antenna).

The radiation resistance of an antenna that emits dipole radiation is proportional to the square of the peak (electric or magnetic) dipole moment of the antenna. This dipole moment is roughly the product of the peak charge times the length of the antenna in the case of a linear (electric) antenna, and is the product of the peak current times the area of the antenna in the case of a loop (magnetic) antenna. Hence, it is hard to increase the radiation resistance of small linear or loop antennas by altering their shapes.[2]

One suggestion for a small antenna is the so-called “crossed-field” antenna [2]. Its proponents are not very explicit as to the design of this antenna, so this problem is based on a conjecture as to its motivation.[3]

It is well known that in the far zone of a dipole antenna the electric and magnetic fields have equal magnitudes (in Gaussian units), and their directions are at right angles to each other and to the direction of propagation of the radiation. Furthermore, the far zone electric and magnetic fields are in phase. The argument is, I believe, that it is desirable if these conditions could also be met in the near zone of the antenna.

The proponents appear to argue that in the near zone the magnetic field $\mathbf{B}$ is in phase with the current in a simple, small antenna, while the electric field $\mathbf{E}$ is in phase with the charge, but the charge and current have a 90° phase difference. Hence, they imply, the electric and magnetic fields are 90° out of phase in the near zone, so that the radiation (which is proportional to $\mathbf{E} \times \mathbf{B}$) is weak.

The concept of the “crossed-field” antenna seems to be based on the use of two small antennas driven 90° out of phase. The expectation is that the electric field of one of the

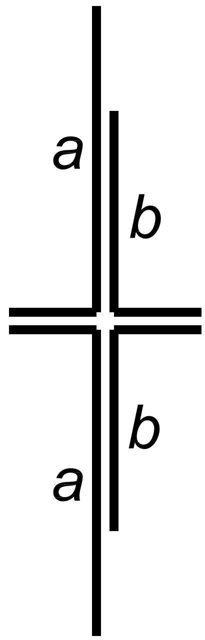
[1] A center-fed linear dipole antenna of total length $l \ll \lambda$ has radiation resistance $R_{\rm linear} = (l/\lambda)^2\, 197\,\Omega$, while a circular loop antenna of diameter $d \ll \lambda$ has $R_{\rm loop} = (d/\lambda)^4\, 1948\,\Omega$. For example, if $l = d = 0.1\lambda$ then $R_{\rm linear} = 2\,\Omega$ and $R_{\rm loop} = 0.2\,\Omega$.

[2] That there is little advantage to so-called small fractal antennas is explored in [1].

[3] A variant based on combining a small electric dipole antenna with a small magnetic dipole (loop) antenna has been proposed by [3].

antennas will combine with the magnetic field of the other to produce radiation that is much more powerful than that from either of the two antennas separately.

If suffices to consider two small linear dipole antennas, say of lengths $2a \ll \lambda$ and $2b \ll \lambda$, as shown in the figure below. Discuss the dependence of the total power radiated by the two antennas as a function of a, b, and the drive currents $I_a e^{-i\omega t}$ in antenna a and $I_b e^{-i(\omega t+\phi)}$ in antenna b.



A variant of the "crossed-field" antenna is the so-called "EH" antenna [4, 5, 6],[4] which is a short, linear dipole antenna in which the currents in the two arms are claimed to be driven 90° out of phase.[5,6] Discuss the power radiated by such an antenna.

An important aspect of practical antennas is the behavior of the feed line between the rf power source and the antenna. Ideally this is a two-wire transmission line, such as a coaxial cable, that carries equal and opposite currents on the two wires. Then the radiation from the currents in the two wires cancels and the feed line can be ignored when discussing radiation by the antenna itself.

The pair of dipole antennas that comprise the "crossed-field" antenna can be operated with their drive currents 90° out of phase simply and precisely by connecting them to a pair of coaxial cables from a single rf power source such that one cable is $\lambda/4$ longer than the other. Then each coaxial cable operates as an ideal, nonradiating transmission line and these

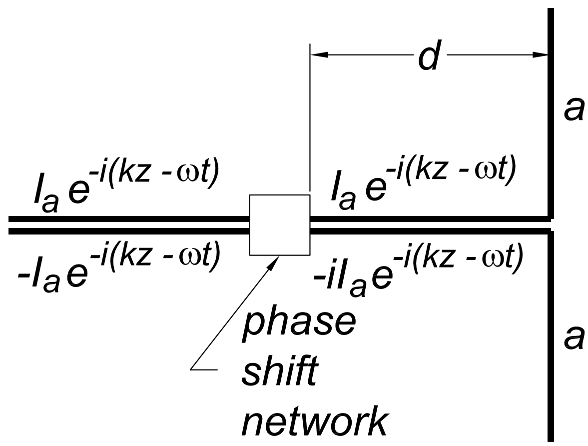
[4]Reference [4] also discusses the so-called "H_z" antenna which consists of a pair of small, loop antennas placed side by side and driven 90° out of phase. The behavior of this antenna pair is essentially the same as that of a "crossed-field" antenna and need not be discussed separately.

[5]In one definition of the EH antenna [7], it is recommended that the antenna include "tuning" coils such that when the system is considered as two-terminal device, the applied voltage would be exactly 90° out of phase with the current. Since this implies that the system would draw zero power, you need not consider this suggestion further.

[6]The EH-Antenna web site has now dropped the claim that the two arms of the antenna are driven 90° out of phase.

lines can be neglected in an analysis of the radiation.[7]

However, an "EH" antenna cannot be driven by a single, nonradiating transmission line in which the currents on its two conductors are opposite rather than 90° out of phase. As shown in the figure below, suppose that a small phase shift network is installed in the transmission line at a distance $d \approx \lambda/4$ from the feed points of an "EH" antenna, such that over the length d the current in one conductor is $I_a e^{-i\omega t}$ and that in the other conductor is $-iI_a e^{-i\omega t}$ (where currents in the figure are positive when flowing to the right and when flowing upwards).



Deduce the contribution to the radiation from the currents in this feed line.

Another aspect of practical reality is that antennas are mounted close to the Earth's surface, which acts like a perfectly conducting ground plane to a reasonable approximation. Discuss the effect of this ground plane on the behavior of the "crossed-field" and "EH" antennas.

2 Solution

2.1 The "Crossed-Field" Antenna

We recall that the time-averaged power P radiated by an antenna system with total electric dipole moment $\mathbf{p} = \mathbf{p}_0 e^{-i\omega t}$ is

$$P = \frac{|\ddot{\mathbf{p}}|^2}{3c^3} = \frac{\omega^4 |\mathbf{p}_0|^2}{3c^3}. \qquad (1)$$

For completeness, we deduce the dipole moment of a center-fed linear antenna of length $2a$. We take the conductors to be along the z-axis, with the feed point at the origin. The current at the feed point is $I_a e^{-i\omega t}$, but it must fall to zero at the tips of the antenna $z = \pm a$. When $a \ll \lambda$ the current distribution can only have a linear dependence on z, so its form

[7] If "crossed-field" antennas had been driven in this manner, the controversy as to their performance might have been settled long ago.

must be[8]

$$I(z,t) = I_a e^{-i\omega t}\left(1 - \frac{|z|}{a}\right). \tag{2}$$

The charge distribution $\varrho(z,t) = \rho(z)e^{-i\omega t}$ along the antenna can be deduced from the current distribution (2) using the equation of continuity (*i.e.*, charge conservation), which has the form $\partial I/\partial z = -\partial\varrho/\partial t = i\omega\rho(z)e^{-i\omega t}$. Thus,

$$\rho(z) = \pm\frac{iI_a}{\omega a}. \tag{3}$$

The electric dipole moment p_a is therefore

$$p_a(t) = \int_{-a}^{a} z\rho(z)e^{-i\omega t}\,dz = \frac{iI_a a}{\omega}e^{-i\omega t}. \tag{4}$$

Similarly, the electric dipole moment of antenna b, whose current is $I_b e^{-i(\omega t+\phi)}$, is

$$p_b(t) = \frac{iI_b b}{\omega}e^{-i(\omega t+\phi)}. \tag{5}$$

The total electric dipole moment for the two antennas of the present example is

$$p(t) = i\frac{I_a a + I_b b e^{i\phi}}{\omega}e^{-i\omega t} \equiv p_0 e^{-i\omega t}. \tag{6}$$

The total (time-averaged) power radiated by the "crossed-field" antenna is, using eq. (1),

$$P = \frac{\omega^4\,|\mathbf{p}_0|^2}{3c^3} = \frac{\omega^2}{3c^3}(I_a^2 a^2 + I_b^2 b^2 + 2I_a a I_b b\cos\phi) = P_a + P_b + 2\sqrt{P_a P_b}\cos\phi, \tag{7}$$

where P_a and P_b are the powers that would be radiated by each antenna in the absence of the other.

2.2 The "EH" Antenna

We take the total length of the antenna to be $2a$ along the z-axis. The drive current is assumed to have the form

$$I(z,t) = I_a e^{-i\omega t}\left(1 - \frac{|z|}{a}\right)\begin{cases} 1 & (0 < z < a), \\ i & (-a < z < 0), \end{cases} \tag{8}$$

which incorporates a 90° phase difference between the currents in the two arms of the antenna. Comparing with eq. (3), we see that the distribution of charge along the antenna has the form

$$\rho(z) = \frac{I_a}{\omega a}\begin{cases} i & (0 < z < a), \\ 1 & (-a < z < 0). \end{cases} \tag{9}$$

[8] Strictly speaking, only the current that is in phase with the drive current must have the form (2). There actually exists a small current that is 90° out of phase with the drive current, and which vanishes at $z = 0$ as well as $z = \pm a$. This current is needed to provide some additional electric field in the near zone such that the tangential component of the total electric field vanishes along the (good) conductors. However, this current does not affect the radiated power, and may be neglected in the present discussion.

The electric dipole moment p_a of the antenna is therefore

$$p_a(t) = \int_{-a}^{a} z\rho(z)e^{-i\omega t}\,dz = \frac{I_a a}{2\omega}(1+i)e^{-i\omega t} \equiv p_0 e^{-i\omega t} \tag{10}$$

The time-averaged power radiated by the "EH" antenna is, using eq. (1),

$$P = \frac{\omega^4 |\mathbf{p}_0|^2}{3c^3} = \frac{\omega^2 a^2 I_a^2}{6c^3} = \frac{P_a}{2}\,, \tag{11}$$

where $P_a = \omega^2 a^2 I_a^2/3c^3$ is the power that would be radiated if the two arms of the antenna were driven in phase.

2.3 Radiation from the Feed Line to the "EH" Antenna

When a feed line contains "unbalanced" currents, as specified in the "EH" antenna scheme, there is radiation from the feed line as well as from the antenna proper. When an "unbalanced" feed line is long compared to the size of the antenna, the radiation from the feed line is much larger than that of the antenna proper. Here, we deduce the radiation from the feed line shown in the figure on p. 3.

We take the lower conductor of the feed line to be the inner conductor of the coaxial cable. Then, the current on the inner conductor is $-iI_a e^{i(kz-\omega t)}$, where $k = 2\pi/\lambda = \omega/c$, and the feed line is taken to lie along the z-axis.[9] Anticipating further analysis in the following section, we suppose the feed line is vertical and extends from $z = 0$ up to $z = d$.

Accompanying the current on the inner conductor is a current $iI_a e^{i(kz-\omega t)}$ on the inside of the outer conductor. These equal and opposite currents are associated with the transverse electromagnetic wave (TEM) that propagates inside the coaxial cable. These two current do not directly produce any radiation in the far zone.[10]

In addition, there is a current $I_o e^{i(kz-\omega t)}$ that flows on the outside of the outer conductor. The radiation from the feed line is due to this current.

Since the total current on the outer conductor is $I_a e^{i(kz-\omega t)}$, we have that

$$I_o e^{i(kz-\omega t)} = I_a e^{i(kz-\omega t)} - (iI_a e^{i(kz-\omega t)}) = (1-i)I_a e^{i(kz-\omega t)}. \tag{12}$$

The radiating current (12) extends over a distance d that is not small compared to a wavelength. In this case, an accurate calculation of the radiation should go beyond the dipole approximation. Fortunately, there is an "exact" prescription for the (time-averaged) angular distribution of radiation in the far zone from a specified time-harmonic current distribution [8],

$$\frac{dP}{d\Omega} = \frac{\omega^2}{8\pi c^3}\left|\hat{\mathbf{k}} \times \left[\hat{\mathbf{k}} \times \int \mathbf{J}(\mathbf{r},t)e^{-i\mathbf{k}\cdot\mathbf{r}}\,d\mathrm{Vol}\right]\right|^2. \tag{13}$$

[9]The speed of light inside the coax is in general less than c, the speed of light in vacuum. We ignore this detail in the present analysis.

[10]The TEM wave inside the coaxial feed line transfers power from the rf source to the antenna, the flow of which is described by the (time-averaged) Poynting vector. Some of the power inside the coax cable is "radiated" into the far zone, and lines of the Poynting vector in the far zone are direct continuations of lines of the Poynting vector inside the cable. From the perspective of power flow, the antenna structure, along with the coaxial feed line, merely "guides" the transmission of rf power from the source into the far zone.

In the present problem $\mathbf{J}(\mathbf{r},t)\,d\text{Vol} \to I_o e^{i(kz-\omega t)}\,\hat{\mathbf{z}}\,dz$. For a far-zone observer at angles (θ,ϕ) in a spherical coordinate system, the unit vector $\hat{\mathbf{k}}$ is given by

$$\hat{\mathbf{k}} = \sin\theta\cos\phi\;\hat{\mathbf{x}} + \sin\theta\sin\phi\;\hat{\mathbf{y}} + \cos\theta\;\hat{\mathbf{z}}. \tag{14}$$

Combining eqs. (12)-(14), we have

$$\frac{dP(\theta,\phi)}{d\Omega} = \frac{\omega^2 I_a^2}{4\pi c^3}\sin^2\theta\left|\int_0^d e^{ikz(1-\cos\theta)}\,dz\right|^2 = \frac{\omega^2 I_a^2 d^2}{4\pi c^3}\sin^2\theta\left[\frac{\sin\frac{kd}{2}(1-\cos\theta)}{\frac{kd}{2}(1-\cos\theta)}\right]^2. \tag{15}$$

The factor in eq. (15) in brackets represents the departure of the radiation pattern from that of an ideal dipole, due to the finite length d of the feed line. The factor is 1 for $\theta = 0^\circ$ and 180° for any value of d. At $\theta = 90^\circ$ the (factor)^2 is 0.81 for $d = \lambda/4$ and 0.41 for $d = \lambda/2$.

The radiation pattern (15) is generated by the traveling-wave current of eq. (12), and which is sometimes called the "Beverage" antenna pattern (US patent 1,381,089, June 7, 1921) [10]. Typical "Beverage" antennas are several wavelengths long to increase their directivity (along the direction of the traveling wave).

For comparison, the radiation pattern of a center-fed linear dipole antenna of total length d is

$$\frac{dP(\theta,\phi)}{d\Omega} = \frac{I^2}{2\pi c}\left[\frac{\cos(\frac{kd}{2}\cos\theta) - \cos\frac{kd}{2}}{\sin\frac{kd}{2}\sin\theta}\right]^2. \tag{16}$$

At $\theta = 90^\circ$ the square of the factor in brackets in eq. (16) is 0.09 for $d = \lambda/4$ and 1.0 for $d = \lambda/2$. For $d < \lambda/2$, the radiation pattern of the "EH" feed line is more strongly peaked in the equatorial plane than is the pattern of a center-fed linear antenna.

We can integrate eq. (15) over angles to obtain the total (time-averaged) power radiated by the "EH" feed line, but there is no simple analytic result. As an approximation, we ignore the factor in brackets, which gives[11]

$$P = \int \frac{dP(\theta,\phi)}{d\Omega}\,d\Omega \approx \frac{2\omega^2 d^2 I_a^2}{3c^3}. \tag{17}$$

When $d \gg a$ this is large compared to the power $P_a = \omega^2 a^2 I_a^2/6c^3$ radiated by the arms of the antenna, as found in eq. (11). Hence, with the phase shift network located at a distance $d \gg a$ from the antenna, as recommended in [5], most of the radiation of an "EH" antenna actually comes from the feed line between the phase shift network and the antenna, rather than from the antenna itself.[12]

2.4 Effects of the "Ground" Plane

The discussion thus far has presumed that the antennas are located in free space, far from any other conductors. But in many cases antennas are arrayed a small distance above the surface of the Earth, which acts like an ideal ground plane to some approximation.

[11] Equation (17) is obtained in the approximation that the length d of the feed line is small compared to the wavelength λ. In this approximation the current $I_o e^{-i\omega t}$ has no spatial dependence. Then, no charge accumulates along the feed line itself. Rather, charges $\pm Q = \pm i I_o/\omega$ accumulate at the two ends of the feed line. The electric dipole moment of these charges is Qd, from which eq. (17) could also be obtained.

[12] An empirical study of radiation from the feed line of an "EH" antenna is given in [11].

Then, all charges in the system can be thought of as having "image" partners of the opposite sign located at distances underground equal to the heights of the actual charges above ground. The oscillating "image" charges create radiation (actually due to currents on the surface of the Earth), that interferes with the radiation from the nominal antenna.

One option is to build only one, vertical arm of an antenna, and connect the "return" conductor of the feed line to "ground" directly below that single arm. Then the "image" of the single arm acts like the second arm of a dipole antenna, whose behavior is like that of the antennas modeled here to the extent that the Earth is a perfect conductor. The "crossed-field" antenna of [2] is based on a single physical arm plus "image" arm. Hence, its behavior is reasonably well modeled by the analysis of sec. 2.1.

The "EH" antenna of [5] is stated to be mounted with its center at distance $d = \lambda/8$-$\lambda/4$ above the Earth's surface. The dipole moment of the charges on the conductor of the antenna has an "image" dipole of the opposite sign located at distance d underground. The radiation patterns of the antenna dipole and its "image" interfere constructively if $d = \lambda/4$ [12], but the radiation from the antenna dipole is negligible compared to that from the feed line if $d \gg a$.

Presumably the phase shift network is on the ground at the base of the antenna tower, such that the feed line runs upwards from $z = 0$ to $z = d$. The current $I_o e^{i(kz-\omega t)}$ on the outside of the feed line creates an "image" current of the same form. That is, the "image" of an upward moving positive charge is a downward moving negative charges, and both of these motions corresponds to upward currents. The (time-averaged) radiation pattern of the feed line plus its image can be calculated as in eq. (15),

$$\frac{dP(\theta,\phi)}{d\Omega} = \frac{\omega^2 I_a^2}{4\pi c^3} \sin^2\theta \left| \int_{-d}^{d} e^{ikz(1-\cos\theta)}\, dz \right|^2 = \frac{\omega^2 I_a^2 d^2}{\pi c^3} \sin^2\theta \left[\frac{\sin(kd(1-\cos\theta))}{kd(1-\cos\theta)} \right]^2 . \qquad (18)$$

The total (time-averaged) radiated power, neglecting the factor in brackets, is approximately

$$P \approx \frac{8\omega^2 d^2 I_a^2}{3c^3} \equiv \frac{I_a^2}{2} R_{\rm rad} , \qquad (19)$$

where the radiation resistance is

$$R_{\rm rad} = \frac{16\omega^2 d^2}{3c^3} = \frac{64\pi^2 d^2}{3\lambda^2 c} \approx 6300 \frac{d^2}{\lambda^2}\, \Omega, \qquad (20)$$

noting that $1/c = 30\ \Omega$. For $d = \lambda/8$ the radiation resistance is about 100 Ω.

The length a of the conductors of the "EH" antenna plays no role in this result, and these conductors could be shortened to zero length with no change in the performance of the system, whose radiation is due to the feed line, and the antenna itself.[13]

3 Comments

Equation (7) tells us that a "crossed-field" antenna pair would actually work better if both antennas were driven in phase than if they are driven 90° out of phase ($\phi = 90°$) as recom-

[13]The nominal "EH" antenna does play a role as part of the phase-shift network, the behavior of the system would be different if the nominal antenna were removed.

mended by its proponents.[14]

Similarly, eq. (11) tells us that an "EH" antenna (driven at its terminals without a feedline) would work better if both arms of the antenna were driven in phase. Equations (15) and (17) indicate that the power radiated the "EH" antenna system considered here would be largely due to the unbalanced currents in its feedline. This "EH" antenna system is better described as a traveling-wave antenna of the type introduced by Beverage in 1921 [10].

As-built EH antennas [6] use arms that are cylindrical sleeves, one of which surrounds the coaxial feedline and provides good capacitive coupling to the outside the outer conductor of the coax. This configuration leads to strong currents on the outside of the feedline, which then acts as the principal radiator of the antenna [11]. See [13] for a systematic discussion of how one can achieve good antenna performance by controlled generation of currents on the outside of a coaxial cable.

A Appendix: Model of an "As-Built" EH Antenna

The discussion in secs. 2.2-3 followed some of the apparent descriptions of EH antennas [5], but some of the "as-built" systems [6, 11] are simpler. In this Appendix we consider a variant in which the rf transmitter is at ground level, connected to a vertical, coaxial transmission line of height d, to which is appended a "matching network", and two coaxial sleeves of length much less than d. The model is that the currents in the coaxial sleeves (*i.e.*, the nominal EH antenna) play little role in the radiation of the antenna (and will be ignored henceforth), but the capacitance between the sleeves and the outside of the outer conductor of the coaxial cable permits currents to flow on the cable. Thus, the model is of an inverted monopole antenna of length d.

The "matching network" is tuned such that the terminal impedance of the nominal antenna is 50 Ω, but the currents in this network make negligible contribution to the radiation.

A simple model for the current on the outside of the coaxial cable is

$$I(0 < z < d, t) = I_0 \frac{\sin kz}{\sin kd} e^{-i\omega t}. \tag{21}$$

If there is no ground current, then the radiation pattern can be calculated from eq. (13) as

$$\begin{aligned}
\frac{dP(\theta,\phi)}{d\Omega} &= \frac{\omega^2 I_0^2}{4\pi c^3} \frac{\sin^2\theta}{\sin^2 kd} \left| \int_0^d \sin kz\, e^{-ikz\cos\theta)}\, dz \right|^2 \\
&= \frac{\omega^2 I_0^2}{16\pi c^3} \frac{\sin^2\theta}{\sin^2 kd} \left| \int_0^d e^{ikz(1-\cos\theta)}\, dz - \int_0^d e^{-ikz(1+\cos\theta)}\, dz \right|^2 \\
&= \frac{I_0^2}{4\pi c} \frac{\left|1 - (\cos kd + i\cos\theta \sin kd)\, e^{-ikd\cos\theta}\right|^2}{\sin^2\theta \sin^2 kd} \\
&= \frac{I_0^2}{4\pi c} \frac{[\cos(kd\cos\theta) - \cos kd]^2 + [\cos\theta \sin kd - \sin(kd\cos\theta)]^2}{\sin^2\theta \sin^2 kd}.
\end{aligned} \tag{22}$$

[14]This comment is also made in [9].

If the antenna is above an ideal ground plane, and the current on the outside of the coax goes to zero at its base ($z = 0$), then the image current is

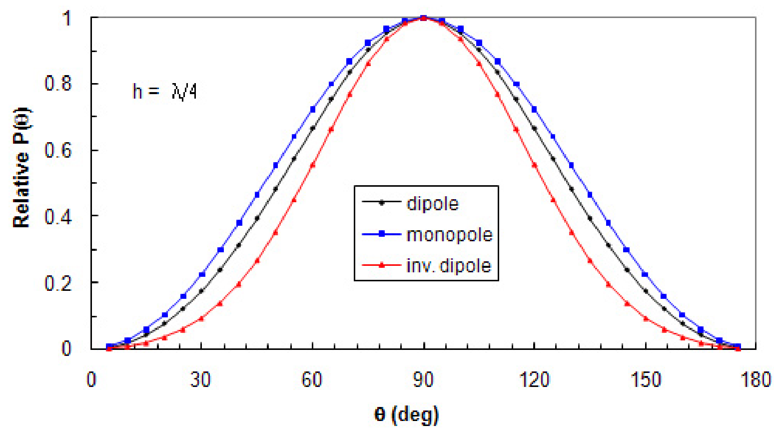
$$I(-d < z < 0, t) = -I_0 \frac{\sin kz}{\sin kd} e^{-i\omega t}, \tag{23}$$

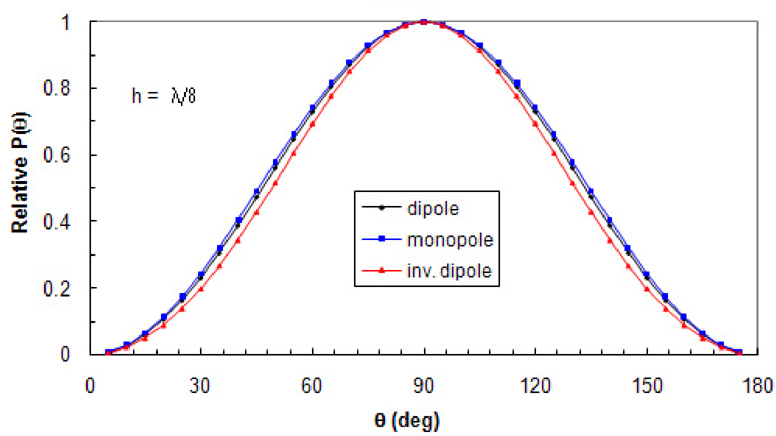
and the radiation pattern is given by

$$\begin{aligned} \frac{dP(\theta,\phi)}{d\Omega} &= \frac{\omega^2 I_0^2}{4\pi c^3} \frac{\sin^2\theta}{\sin^2 kd} \left| \int_0^d \sin kz\, e^{-ikz\cos\theta)}\, dz - \int_{-d}^0 \sin kz\, e^{-ikz\cos\theta)}\, dz \right|^2 \\ &= \frac{I_0^2}{\pi c} \frac{|1 - \cos kd \cos(kd\cos\theta) - \cos\theta \sin kd \sin(kd\cos\theta)|^2}{\sin^2\theta \sin^2 kd}. \end{aligned} \tag{24}$$

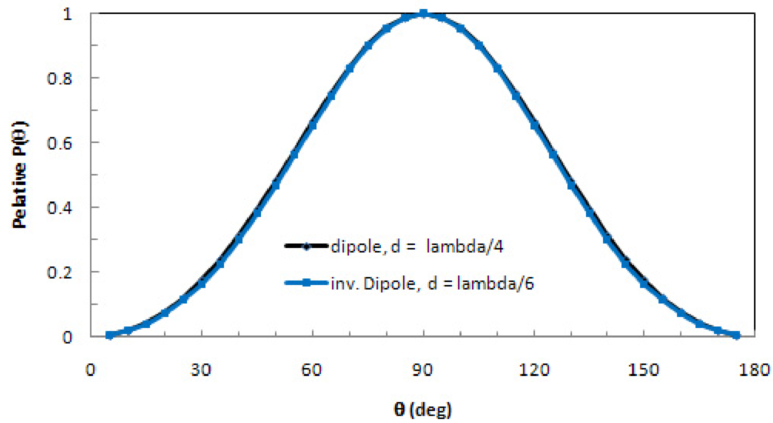
The case of an inverted monopole antenna over a ground plane could be called an inverted dipole in the sense that the currents in the real and image arms are maximal at the ends ($z = \pm d$) and zero at the center ($z = 0$).

The radiation patterns for an ordinary dipole of half-height d, of a monopole of height d according to eq. (22), and of an of an inverted dipole of half-height d according to eq. (24) are shown below for $d = \lambda/4$ and $d = \lambda/8$.





Of possible interest is the fact that the radiaton pattern of an ordinary half-wave dipole ($d = \lambda/4$) is extremely similar to that of an inverted dipole with $d = \lambda/6$, as shown in the figure on the next page. Also, the radiation pattern of a monopole antenna (inverted or not) with $d = \lambda/2$ is essentially identical to that of an ordinary half-wave dipole.



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