

# What is $i^i$ ?

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*This amusing problem is posed by R.C. Henry at <http://henry.pha.jhu.edu/rch.html>*

Henry's solution invokes the facts that  $(a^b)^c = a^{bc}$  and that one can write  $i = e^{(4n+1)i\pi/2}$ , for any real integer  $n$ , to find the infinite set of answers:

$$i^i = (e^{(4n+1)i\pi/2})^i = e^{-(4n+1)\pi/2}. \quad (1)$$

Henry favors setting  $n = 0$ ,  $\Rightarrow i^i = e^{-\pi/2} \approx 0.208$ .

Following Henry's advice to try the problem before looking at his solution, I proceeded differently:

$$i^i = x = e^{\ln x} = e^{-(4n+1)\pi/2}, \quad (2)$$

as in eq. (1), using

$$\ln x = i \ln i = i \ln(1 e^{i\pi/2}) = i[0 + (4n + 1)i\pi/2] = -(4n + 1)\pi/2, \quad (3)$$

which follows from the definition of the logarithm of a complex number,

$$\ln z = \ln r + i(\phi + 2n\pi), \quad \text{where} \quad z = r e^{i\phi}. \quad (4)$$