Mertz' Paramagnetic Fluid Pump

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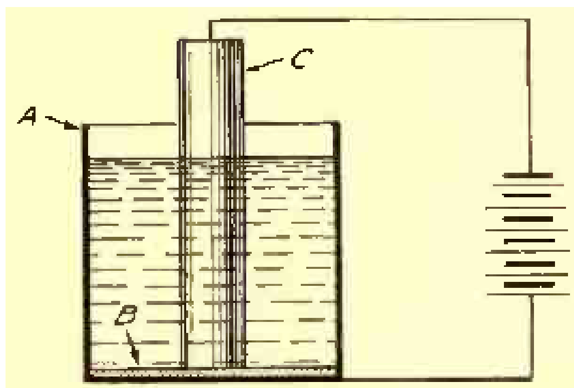
1 Problem

Discuss the principle of operation of an electromagnetic "pump" described by Mertz (1915) in a popular electrical magazine [1].

As sketched below, an annular volume of copper sulfate and water, a conducting, paramagnetic fluid,[1] surrounds a strong, cylindrical, conducting, permanent magnet C that rests on an insulating base B, and the liquid is surrounded by an outer conducting cylinder A. When a strong DC current flows between the outer cylinder and the cylindrical magnet, the copper-sulfate solution rotates azimuthally (and the current flows on spiraling paths in the liquid). The direction of rotation depends on the sign of the electric current.



2 Solution

2.1 J × B Force

The most common type of electromagnetic pump is based on the $\mathbf{J} \times \mathbf{B}$ force density in a current-carrying liquid,[2] which latter is typically nonmagnetic.

In Mertz apparatus, if we approximate the current in the liquid as flowing in horizontal planes, the azimuthal component of the azimuthal force on the liquid. force density would be $-J_r B_z$, in a cylindrical coordinate system (r, ϕ, z) with the z-axis along that of the cylinder. If the conducting cylinder C were nonmagnetic, there would be no z-component to the magnetic field (due only to the electric current driven by the battery). Hence, it is essential that cylinder C be a permanent magnet for there to be an azimuthal force on the liquid.

[1]Mertz' paper is one of the earliest to mention magnetic fluids. In recent years, so-called ferrofluids have found considerable application. See, for example, [2].

[2]This effect was perhaps first demonstrated by Hering [3], and analyzed by Northrup [4] (1907).

However, the empirical evidence is that if the liquid were nonmagnetic, the azimuthal force density in the liquid would be too small to produce azimuthal rotation for practical currents and strengths of the permanent magnet.

2.2 Force on Paramagnetic Ions

Instead, the azimuthal flow in Mertz' example is largely due to the magnetic force on the paramagnetic copper ions in the liquid.

To deduce this force, we model the magnetic moment $\mathbf{m}$ of an ion as two equal an opposite (effective) magnetic charges p separated by small distance $\mathbf{d}$, *i.e.*, $\mathbf{m} = p\,\mathbf{d}$. The force on an effective magnetic charge p at position $\mathbf{x}$ in a magnetic field $\mathbf{B}$ is $\mathbf{F} = p\,\mathbf{B}(\mathbf{x})$.[3] Then, the force on a magnetic dipole $\mathbf{m}$ with $-p$ at $\mathbf{x}$ and p at $\mathbf{x} + \mathbf{d}$ is,

$$\mathbf{F} = \lim_{\mathbf{d}\to 0,\ p\,\mathbf{d}=\mathbf{m}} [p\,\mathbf{B}(\mathbf{x}+\mathbf{d}) - p\,\mathbf{B}(\mathbf{x})] = p(\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{B}(\mathbf{x}) = (\mathbf{m}\cdot\boldsymbol{\nabla})\mathbf{B}. \tag{1}$$

Further, we suppose that the paramagnetic ions in the liquid are aligned (on average) with the magnetic field, such that,

$$\mathbf{m} = k\mathbf{B}. \tag{2}$$

for a (temperature-dependent) constant k. Hence, the force on a magnetic dipole in the liquid is,[4]

$$\mathbf{F} = k(\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B}. \tag{3}$$

In a cylindrical coordinate system (r, ϕ, z) with the z-axis along that of the cylinder, the permanent magnetic field has r- and z-components, while that due to the electrical current in the conducting cylinder C has only a ϕ-component. Both of these fields are azimuthally symmetric.[5]

In general, the magnetic force on the liquid has all three of r-, ϕ- and z-components, but we emphasize only the ϕ component here,

$$F_\phi = k\left(B_r(r,z)\frac{\partial}{\partial r} + B_z(r,z)\frac{\partial}{\partial z}\right)B_\phi(r,z). \tag{4}$$

The azimuthal force (4) is nonzero only if there exists both the field $(B_r, 0, B_z)$ due to the permanent magnet C and the field $(0, B_\phi, 0)$ due to the electrical current through the magnet.

Note that if the magnet C were very long, and the liquid only near its midplane in z, then B_r would be negligible, B_ϕ would depend only on r, and the azimuthal force would be negligible.

[3]See Appendix A of [5] for discussion of the force on effective magnetic charges associated with magnetic materials that actually are based on Ampèrian "molecular currents".

[4]In general, $\boldsymbol{\nabla}B^2 = 2(\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B} + 2\mathbf{B}\times(\boldsymbol{\nabla}\times\mathbf{B})$. If there were no current in the liquid (and it were in a steady state), $\boldsymbol{\nabla}\times\mathbf{B} = 0$, and we could write the force as $\mathbf{F} = k\boldsymbol{\nabla}B^2/2$. Then, with $\mathbf{B}$ independent of ϕ, as in the present example, there would be no azimuthal force.

[5]The spiraling current in the liquid contributes to all three of B_r, B_ϕ and B_z. This effect must be included in a detailed calculation; otherwise $F_\phi = 0$ as remarked in the preceding footnote.

References

[1] P. Mertz, *Interesting Experiments in Electricity*, Electrical Experimenter **3**, 7 (1915),
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[2] R.E. Rosensweig, *Ferrohydrodynamics* (Cambridge U. Press, 1985).

[3] C. Hering, *A Practical Limitation of Resistance Furnaces; The "Pinch" Phenomenon*, Trans. Am. Electro. Soc. **11**, 329 (1907),
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[4] E.F. Northrup, *Some Newly Observed Manifestations of Forces in the Interior of an Electric Conductor*, Phys. Rev. **24**, 474 (1907),
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[5] K.T. McDonald, *Poynting's Theorem with Magnetic Monopoles* (March 23, 2013),
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