

8 Friction in Fluids

Introduction

‘Modern’ physics began to flourish in the 1600’s when it was realized that the basic laws of motion were most clearly demonstrated in situations with minimal friction. Correspondingly, the motion of fluids would be simpler to understand if they were friction free. However, all real fluids have intrinsic friction due to two effects: the weak attraction between fluid molecules (viscosity), and the transfer of momentum from fluid molecules that bounce off objects (high-speed drag). In this laboratory we consider three situations in which fluid friction is important:

1. Flow of fluid in a circular pipe,
2. Slow linear motion of objects through a fluid.
3. Levitation by an air jet.

The first two are examples of viscous drag, while the third emphasizes high-speed drag.

Viscous Drag

Quantitative consideration of friction in fluids began with Newton, whose original example concerned two plates of area A separated by a distance Δy in a fluid, as sketched in Fig. 1.

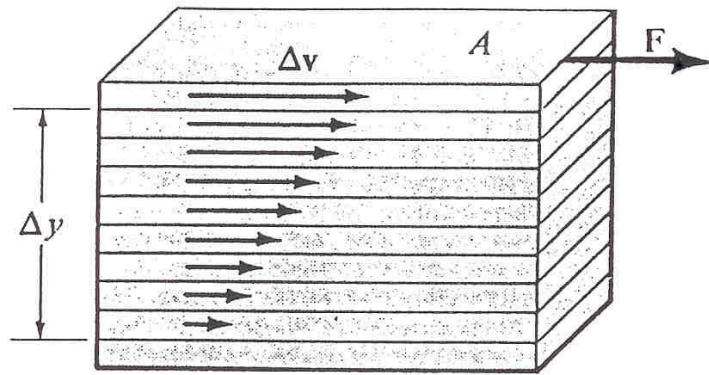


Figure 1: Fluid flow between two plates of area A , separation Δy and relative velocity v .

A basic insight is that fluid which is close to a solid surface has the same velocity as that surface due to the friction of the fluid. If one plate has velocity Δv relative to the other as shown, the velocity of the fluid between the plates varies from 0 to Δv and a force F is required to maintain the motion. The ratio of ratios

$$\eta = \frac{F/A}{\Delta v/\Delta y} \quad (1)$$

is an intrinsic property of the fluid and is called the viscosity. The corresponding expression for the viscous drag force is

$$F_{\text{viscous}} = \eta A \frac{\Delta v}{\Delta y} \quad (2)$$

High-Speed Drag

When an object moves through a fluid with velocity v , presenting area A to the fluid, the molecules that bounce off the object transfer momentum to the object at rate $\Delta p/\Delta t = F$, where F is the resulting drag force. An estimate of this force is quickly obtained by noting that in time Δt a volume of fluid $V = Av\Delta t$ hits the object. The momentum carried in this volume is $\rho_0 V v$ where ρ_0 is the mass density of the fluid. If all of this momentum were transferred to the object the corresponding force would be $F = \Delta p/\Delta t = \rho_0 A v^2$.

This argument is not precise, and it is customary to include a dimensionless drag coefficient C_D in the definition, as well as a factor of $1/2$:

$$F_{\text{high-speed}} = \frac{C_D}{2} \rho_0 A v^2. \quad (3)$$

The Reynolds Number

The relative importance of high-speed drag and viscous drag is characterized by the (dimensionless) Reynolds number N_R , which is just their ratio

$$N_R = \frac{F_{\text{high-speed}}}{F_{\text{viscous}}} \approx \frac{\rho_0 A v^2}{\eta A \Delta v / \Delta y} = \frac{\rho_0 v L}{\eta}. \quad (4)$$

In this we have identified Δv , the change in velocity relevant to viscosity, with the relative velocity v between the object and the fluid. We also replace Δy by L , representing a characteristic length of the object perpendicular to the direction of fluid flow.

Situations with Reynolds number less than one are dominated by viscous drag, while for $N_R > 1$ high-speed drag is more important. We see that the Reynolds number increases with velocity, justifying the name ‘high-speed drag’ for fluid friction at large N_R .

8.1 Viscous Drag at Low Reynolds Number

8.1.1 Flow of Viscous Fluid in a Circular Pipe

An important situation in which fluid viscosity plays a dramatic role is the flow through pipes. The flow rate is claimed to vary as the fourth power of the radius of the pipe (Poiseuille, 1840).

We first sketch an argument why this is so based on dimensional analysis. The flow rate, which we shall call ϕ , is the volume of fluid passing any cross section of the pipe per second. The flow occurs because there is a pressure difference ΔP between the two ends of the pipe whose length is l . The flow will be slower for higher viscosity and longer pipes, and will be greater for pipes of larger radius. We make the ‘educated guess’ that the flow rate varies like

$$\phi \propto \frac{\Delta P R^n}{\eta l}. \quad (5)$$

That is, we guess that the flow is proportional to the pressure drop and inversely proportional to the viscosity and to the length. Our only uncertainty is how the flow rate depends on the radius of the pipe.

Flow rate has dimensions $length^3/time$, pressure has dimensions $mass/(length \times time^2)$ and viscosity has dimensions $mass/(length \times time)$ according to eq. (1). Hence the dimensions of eq. (5) tell us

$$\left[\frac{length^3}{time} \right] = \frac{\left[\frac{mass}{length \times time^2} \right] [length^n]}{\left[\frac{mass}{length \times time} \right] [length]} = \left[\frac{length^{n-1}}{time} \right]. \quad (6)$$

Hence we infer that $n = 4$ as claimed. A full derivation of the flow rate was first made by Stokes (1845) who showed that

$$\phi = \frac{\pi \Delta P R^4}{8 \eta l}. \quad (7)$$

You will study the flow rate of a heavy machine oil in plastic tubes of various radii. These tubes are attached to the base of a vertical cylinder containing oil to a height h above the inlets of the tubes. Hence the pressure difference between the inlet and outlet of the tubes is $\Delta P = \rho g h$, where ρ is the density of the oil.

The density of the oil can be measured by before-and-after weighings of the graduated cylinder in which you collect the oil flow.

Remove the cork from one of the plastic tubes and collect the oil flow in a graduated cylinder. The tube should be horizontal to avoid the effect of gravity on the flow in the tube. The graduated cylinder should be vertical, and the oil should not run down its wall. Use a stopwatch (or Keyboard Timer Modes of program pt) to measure the time t required to collect a volume V of fluid. Recork the tube, and repeat the process for the other two tubes. For example, collect 20 ml with the largest tube, 10 ml with the midsized tube and 5 ml with the smallest tube. The flow will be quite slow!

Measure the height h of the oil in the vertical cylinder above the inlet to the plastic tubes. Since this height decreases as the oil flows out, measure h before and after collecting oil from each tube, and use the average height in later analysis.

Notice how the vertical flow between the mouth of the tube and the surface of the oil in the collecting cylinder is smooth until just above the surface. Then the flow becomes very turbulent and injects numerous small bubbles into the gathering pool of oil. The details of fluid motion can be very complex!

When oil has been collected from all three tubes, weigh the graduated cylinder to determine the density of the oil. Then pour the oil back into the vertical cylinder.

According to the manufacturer the tubes have radii $R = 0.37, 0.307$ and 0.242 cm.

The first step in the analysis is to verify the dependence of flow rate ϕ on pipe radius R . Poiseuille tells us that $\phi \propto R^4$. Fit your flow rate data to the form

$$\phi = K R^n, \quad (8)$$

to see how well $n = 4$ describes your results.

Create a New Sheet Document with Statmost and enter your data on radius in column A (in order of increasing radius) and flow rate in column B. Select Analyses, then Quasi-Linear;

in the pop-up window select Power, Save Options: Calculated Data and X as A (radius) and Y as B (flow). Print the \$RES000n.TXT summary sheet; the Regression Coefficient is the fitted value of n and the Standard Error of B is the error estimate.

The program actually has taken the logarithms of ϕ and R and fit them to a straight line: eq. (8):

$$\log \phi = \log K + n \log R. \tag{9}$$

It is useful to make a log-log plot of your data, which will display this linear dependence of the logarithms. Activate your spreadsheet window, then Plot, then 2D Curve, then Scatter/Lines. Type in an appropriate Title. To plot the data points select A (radius) as X as and B (flow) as Y and deselect all 3 Options; then click on Add Curve. To add the fitted curve, select A as X, D: Dep as Y and Option as Smooth Curve; then click on Add Curve, and on OK to produce the graph. The graph still has linear axes; to change to logarithmic axes, double click on the left vertical axis to bring up the window 2D Axis Properties. Select Axis Mode: Log (Base 10); for Axis Range set Minimum: 0.01 and Maximum: 0.1 (or other more appropriate powers of 10); for Tick Option set Major Num: 2 (if you wish to span only one power of 10) and Minor Num: 8; then click OK. Double click on the bottom axis to convert it to logarithmic. To add grid lines, double click on the top or right frame around the graph, and select X-On and Y-On of Minor Grids. It is advantageous to make the length of one decade the same on the horizontal and vertical axes: from the Draw menu select Move/Resize and enter appropriate values for Width and Height. Before you Print your graph you may want to select Page Setup and deactivate Show Page Footer.

An example is shown in Fig. 2.

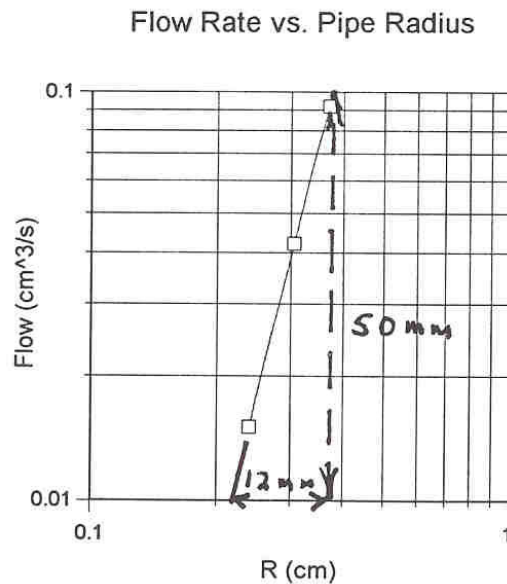


Figure 2: Log-log plot of flow ϕ versus radius R in circular pipes. The slope of the straight-line fit is the power n in the relation $\phi = KR^n$. In this example the fit slope is 4.19 and the slope read off the graph is 50 mm/12 mm = 4.17.

Your data can be further analyzed to extract the viscosity coefficient η . Equation (7) can be rearranged as

$$\eta = \frac{\pi\rho ghR^4}{8l\phi}. \quad (10)$$

Calculate the viscosity η from the data on oil flow in each of the three plastic tubes. Measure length l of each tube and add 2.7 cm for the length inside the large or medium brass fittings or 1.9 cm for the small brass fitting.

8.1.2 Linear Motion of Spheres in a Viscous Fluid

In this part you will explore the viscous motion of small spheres falling through heavy machine oil. Gravity provides the force that will balance the viscous drag described by eq. (2).

An approximate analysis is straightforward, but a detailed proof is one of the more complex exercises in classical physics. A sphere of radius R has surface area $4\pi R^2$ exposed to viscous drag. When the sphere has velocity v the fluid near the sphere takes on the same velocity, but fluid approximately distance R away from the sphere have velocity near zero. Equation (1) tells us that the drag force is

$$F = \frac{\eta A \Delta v}{\Delta y} \approx \frac{4\pi\eta R^2 v}{R} = 4\pi\eta Rv. \quad (11)$$

The detailed theoretical analysis by Stokes (1851) showed that our approximate result should be modified by a factor of 3/2 to be

$$F = 6\pi\eta Rv \quad (\text{Stokes}). \quad (12)$$

[It turns out that eq. (11) is the correct form for drag on an air bubble rising through a liquid.]

The drag force of eq. (12) balances the force of gravity. However, the effective force of gravity is not $mg = (4/3)\pi R^3 \rho g$ where ρ is the density of the sphere. According to Archimedes' principle the fluid provides a buoyant force equal to the weight of the displaced fluid, $(4/3)\pi R^3 \rho_0 g$, where ρ_0 is the density of the fluid. Hence the net force due to gravity is $(4/3)\pi R^3 (\rho - \rho_0)g$. Equating this to eq. (12) the viscosity of the fluid is

$$\eta = \frac{2R^2(\rho - \rho_0)g}{9v} = \frac{D^2(\rho - \rho_0)gt}{18h}, \quad (13)$$

where D is the diameter of the sphere, and t is the time for it to fall height h through the fluid.

Measure the times t for three lead spheres to fall through a column of heavy machine oil. Before dropping the spheres into the oil, measure their diameters with a micrometer; take an average if the 'sphere' is not really spherical. The density of lead is 11.7 g/cm^3 . Use the value for the density ρ_0 of the oil that you found in part 8.1.1.

Calculate the viscosity coefficient η according to eq. (13) and compare with the value found in part 8.1.1. Calculate the Reynolds number (eq. (4)) for one of the spheres in the oil.

8.2 Levitation in an Air Jet at High Reynolds Number

An intriguing advertisement in hardware stores in summertime consists of a beach ball levitated above a room fan that blows air upwards. It is plausible that the upward drag of the air jet on the ball counteracts the downward pull of gravity. More impressive is that the ball is not blown sideways off the fan.

[Skydiving scenes in movies are often performed by levitating the actors above giant fans.]

Recall that Bernoulli's equation,

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}, \quad (14)$$

tells us that the pressure is low where the fluid velocity is high. Thus, in the center of the jet where the velocity is high the pressure is low. As a result there is a 'pressure-gradient' force that pushes the ball towards the center of the jet, as well as downwards towards its mouth. For stable levitation the inward pressure-gradient force must be larger than the outward high-speed drag force – even though the upward high-speed drag force overcomes both gravity and the downward pressure-gradient force. Nature kindly permits all of these conditions to be met. Try it!

In this part of the lab you will analyze the velocity of the air jet as a function of height, and measure the drag coefficient C_D of levitating styrofoam spheres. As a simplification you can ignore the vertical pressure-gradient force, which is a good approximation when the sphere levitates more than a diameter above the mouth of the jet.

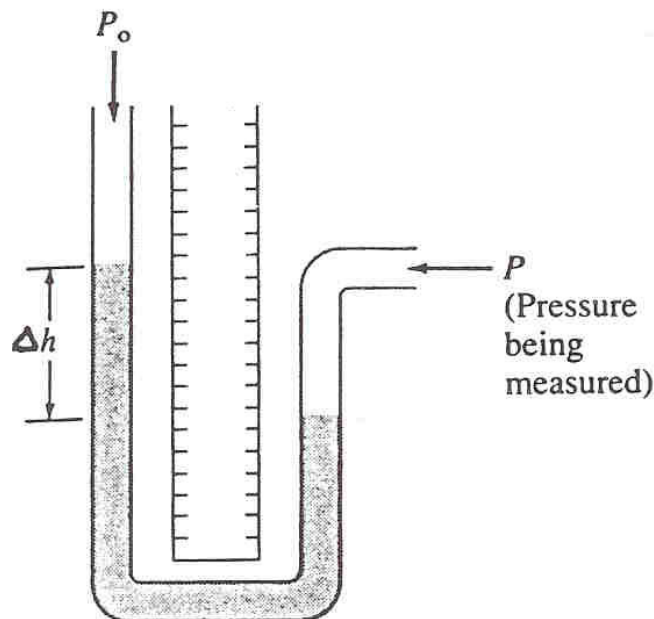


Figure 3: A U-tube manometer to measure the difference in air pressure between the its two inlets.

Aim an air-track blower upwards to create the jet. Levitate as many of the six styrofoam spheres as you can, and record the height of the center of the sphere above the mouth of the jet when successful. The jet must be very vertical to levitate the smaller balls. Measure the diameters d of the spheres and weigh the largest to deduce the density of styrofoam:

$$\rho_{\text{sphere}} = \frac{M_{\text{large}}}{\pi d_{\text{large}}^3 / 6}. \quad (15)$$

Use a water-filled U-tube manometer (Fig. 3) to measure the pressure along the jet axis over the range of heights at which you could levitate spheres. Record the difference Δh between the liquid levels in the left and right arms of the U-tube for heights h in 5-cm steps. Is the pressure at the center of the jet greater or lower than atmospheric pressure?

According to Bernoulli's equation the pressure difference between the center of the jet and still air is

$$\Delta P = \rho_{\text{water}} g \Delta h = \frac{1}{2} \rho_{\text{air}} v_{\text{jet}}^2, \quad (16)$$

where v_{jet} is the velocity of the air jet. Hence

$$v_{\text{jet}} = \sqrt{\frac{\rho_{\text{water}}}{\rho_{\text{air}}} 2g\Delta h}. \quad (17)$$

The density of air is $\rho_{\text{air}} = 0.00129 \text{ g/cm}^3$ at STP.

The expected dependence of the velocity of the jet as a function of height h above its mouth can be deduced from conservation of momentum. Assuming the jet forms a cone of angle θ the area of the jet at height h is $A = \pi h^2 \tan^2 \theta$. That is, more air participates in the jet at greater distances from the mouth (under the excellent approximation that the density of air is constant throughout the jet). The total volume of air in the jet that passes height h in time Δt is $V = Av\Delta t$. The momentum of this air is $Mv = \rho_{\text{air}} Vv = \rho_{\text{air}} Av^2 \Delta t = \pi \rho_{\text{air}} h^2 \tan^2 \theta v^2 \Delta t$. But the total momentum of the moving air must be that same at any height h since there is no (significant) external force on the jet air once it leaves the mouth. Hence we infer that

$$v_{\text{jet}} = \frac{K}{h}. \quad (18)$$

Finally, assuming the weight of the levitating sphere is entirely balanced by the high-speed drag force, eq. (3), we have

$$M_{\text{sphere}} g = \frac{\pi \rho_{\text{sphere}} d^3 g}{6} = \frac{C_D}{2} \rho_{\text{air}} A_{\text{sphere}} v_{\text{jet}}^2 = C_D \frac{\pi \rho_{\text{air}} d^2 v^2}{2}. \quad (19)$$

This can be rearranged to isolate the drag coefficient:

$$C_D = \frac{\rho_{\text{sphere}} g d}{\rho_{\text{air}} 6v^2}. \quad (20)$$

The numerical data analysis can be carried out with the aid of program StatMost. First process the U-tube data to fit the jet velocity to the form $v = c_0 + c_1/h$ as suggested by eq. (18). Create a New Sheet Document and enter the heights h in column A and the

differences Δh in column B. Use Data, then Transform, then Simple Math to calculate the air velocity according to eq. (17) via Formula: $V = \text{SQRT}(1519400 * B)$, assuming Δh is in cm. Create a new column containing $1/h$ via Formula: $H1 = 1/A$. Then use Polynomial Regression of Order: 1 to perform the fit of V to $H1$. Print and Plot as usual.

Create a second spreadsheet containing the diameter d of the spheres in column A and the levitation height h in column B. Create a new column C containing the jet velocity at height h using the above fit; Formula: $V = c_0 + c_1/B$. Finally, create a new column containing the drag coefficient according to eq. (20) via Formula: $C_D = k * A / C^2$ where $k = \rho_{\text{sphere}} g / 6 \rho_{\text{air}}$.

Also, calculate (using StatMost) the Reynolds number N_R for each sphere according to eq. (4) using diameter d as the characteristic length L and the viscosity of air as $\eta = 0.00018$ cgs units. Plot the drag coefficient against the Reynolds number.

At high Reynolds number the drag coefficient should be slightly less than one. Your drag coefficient for the very large spheres might be small because the area of the jet is less than the area of the ball, so the drag area is less than that assumed in eq. (19).

More thorough studies could be done in a wind tunnel but your experiment with the exhaust from a vacuum cleaner has already revealed the essential physics.