

Noncontact Measurement of the Tension of a Wire

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1 The Problem

A conducting wire of mass m and length L is stretched between two fixed points with an unknown tension T . One could determine the tension by plucking the wire and observing the frequency of the vibration. Analyze the following scheme for noncontact plucking. A magnetic field of strength B is applied transversely to the wire over a length $l \ll L$ around the midpoint of the wire. A very short pulse of total charge Q is passes down the wire, which therefore starts vibrating. The voltage induced between the two ends of the vibrating wire is measured as a function of time and a Fourier analysis yields the various frequencies present.

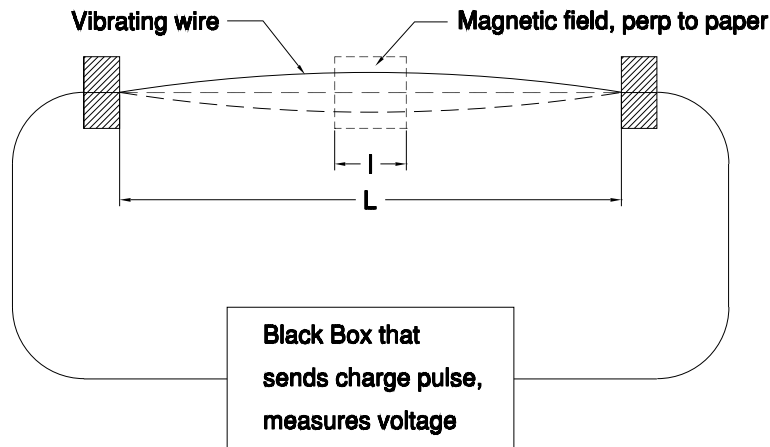


Figure 1: Scheme from noncontact measurement of wire tension.

Calculate the voltage induced at the various frequencies, and relate those frequencies to the tension in the wire. You may ignore damping effects.

A device constructed on the principles of this problem is described in [1].

2 Solution

According to Faraday, the induced voltage is $V = -\dot{\Phi}/c$ in Gaussian units, where Φ is the magnetic flux and c is the speed of light.

We write the amplitude of the transverse oscillation of the wire as $a(x, t)$. The magnetic flux through the circuit containing the wire is $\Phi \approx \Phi_0 + Bla(0, t)$, where Φ_0 is the flux when the wire is at rest. taking the origin at the midpoint of the wire. The boundary conditions

on the wire are $a(L/2, t) = 0 = a(-L/2, t)$, and one of the initial conditions is $a(x, 0) = 0$ at the moment the current pulse is applied. These are sufficient to determine the amplitude as having the form

$$a(x, t) = \sum_{\text{odd } n} a_n \cos \frac{n\pi x}{2L} \sin \omega_n t. \quad (1)$$

The motion satisfies the wave equation $a'' = \ddot{a}/v^2$, where the wave velocity is related to the tension by $v = \sqrt{TL/m}$. Hence,

$$\omega_n = \frac{n\pi v}{2L} = \frac{n\pi}{2} \sqrt{\frac{T}{mL}}. \quad (2)$$

To complete the solution we need another initial condition, corresponding to the transverse impulse due the current pulse interacting with the magnetic field. The Lorentz force on the length l of wire in the magnetic field due to current I is $F = IBl/c$. If this lasts for time Δt the resulting impulse is $\Delta P = I\Delta t Bl/c = QBl/c$. Only a length l of the wire experiences this impulse, so the mass of this section is ml/L and the initial velocity is QBl/mc . In sum, the second initial condition is

$$\dot{a}(x, 0) = \begin{cases} QBl/mc, & |x| < l/2; \\ 0, & |x| > l/2. \end{cases} \quad (3)$$

From the form (1) of $a(x, t)$ we deduce that

$$\dot{a}(x, 0) = \sum_{\text{odd } n} a_n \omega_n \cos \frac{n\pi x}{2L}. \quad (4)$$

Hence, in the usual manner we evaluate the Fourier coefficients as

$$a_n \omega_n = \frac{2}{L} \int_{-L/2}^{L/2} \dot{a}(x, 0) \cos \frac{n\pi x}{2L} dx \approx \frac{2QBl}{mc}, \quad (5)$$

for $l \ll L$.

The induced voltage is then

$$V(t) = -\frac{Bl}{c} \dot{a}(0, t) = -\frac{2Q(Bl)^2}{mc^2} \sum_{\text{odd } n} \cos \omega_n t. \quad (6)$$

The amplitude of the voltage induced at frequency ω_n is $2Q(Bl)^2/mc^2$, independent of n . In practice, the finite values of l and Δt reduce the amplitudes of the higher harmonics.

References

- [1] M.R. Convery, *A Device for Quick and Reliable Measurement of Wire Tension*, (Apr. 29, 1996); <http://puhep1.princeton.edu/~mcdonald/tndc/tension.pdf>