On the Definition of Radiation by a System of Charges

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(September 6, 2010; updated May 16, 2019)

Abstract

Two possible definitions of electromagnetic radiation are,

A. Electromagnetic radiation is the flow of electromagnetic energy described by the Poynting vector,
$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}, \tag{1}$$
in Gaussian units, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, and c is the speed of light in vacuum.

B. Radiation is the flow of energy described by the part of the Poynting vector due to the radiation fields, *i.e.*, those that fall off as $1/r$ from their sources,
$$\mathbf{S}_{\rm rad} = \frac{c}{4\pi}\mathbf{E}_{\rm rad} \times \mathbf{B}_{\rm rad}. \tag{2}$$

Both of these definitions contrast with the "radiation condition" advocated by Sommerfeld that '*the sources must be sources, not sinks of energy. The energy which is radiated from the sources must scatter to infinity; no energy may be radiated from infinity into ... the field*," which leaves the concept of radiation undefined at finite distances from its source(s).

This paper presents several arguments in favor of definition A, of which one of the most powerful is that radiation should be a local property, related only to quantities measurable in the vicinity of an observer, while definition B requires knowledge by the observer of the location of the sources.

A consequence of Definition A (which we advocate accepting) is that radiation is not exclusively associated with accelerated charges (and that not all accelerated charges radiate). Any nonzero Poynting vector is to be called radiation, which implies, for example, that a charge with uniform velocity, and a battery plus resistor, both involve radiation.

1 What is Radiation?

In a broad sense, radiation has come to mean a flow of energy through some medium, possibly vacuum.[1] In a classical view, the energy can be carried by both particles (α and β particle radiation, *etc.*) and by waves (acoustic radiation, electromagnetic radiation, *etc.*). An early view (see, for example, Newton [1]) of optical radiation was that it consists of "rays" which emanate in straight lines from a source.[2] Then, the number of rays crossing any surface

[1] Radiation sometimes has the meaning of the process of emission of energy. Here we take the term "radiation" to describe the result of a process, and not the process itself.

[2] Latin: *radiat-* = emitted in rays; past participle of the verb *radiare*.

enclosing the source is the same, and the number of rays crossing a area element normal to the rays falls off as the square of the distance from the source. A simple model is that optical rays are particles that move with some constant velocity along the path of the ray. The energy carried by the ray is the kinetic energy of the particles.

In the early 1800's Young [2] and Fresnel [3] argued that the optical phenomena of interference and stellar aberration imply that optical rays are actually an aspect of (transverse) waves in an æther. The energy carried by these rays was imagined to be that of the kinetic and potential energy of the undulations of the æther. Maxwell [4] identified optical rays with electromagnetic waves, whose energy is now ascribed to that of the electric and magnetic fields, rather than to a mechanical æther.[3,4]

The concept of rays for waves is only defined on scales larger than a wavelength. A challenge addressed in the present note is to provide an understanding of what can be meant by radiation of electromagnetic energy close to its source(s).

In the quantum theory of electromagnetic fields, they can also be regarded as consisting of particles (photons), and whether their field or particle character is more prominent depends on details of the experiments devised to ascertain that character. This raises the question as to whether Maxwell's equations for electromagnetic fields can lead to results of a particle-like character when considering electromagnetic radiation. Also, in the quantum view, photons have an extent at least that of a characteristic wavelength, and the flow of energy of these photons is not as highly localized as is assumed to be possible in a classical description. Hence, a classical description of the flow of energy close to charges and currents is expected to have finer detail than that possible in the quantum view. For example, lines of classical energy flow in Young's double-slit experiment pass through only one slit or the other, while the quantum view of the resulting interference pattern for a single photon is that the photon has a probability amplitude to pass through both slits.[5] A possible lesson is that one should not worry about details of classical energy flow close to matter. Nonetheless, the present note takes on this challenge.

1.1 Local *vs.* Global Concepts of Radiation

If radiation is to be regarded as a physical concept, then it should be detectable with a suitable device. In particular, such a device should be able to measure the amount (and direction) of the radiation at a particular point in space, averaging over some time interval.

That is, from an operational perspective, radiation should be a local concept. Indeed, a device that measures radiation should not require any knowledge of its source, but should simply report the amount of radiation measured at the observation point.

However, there is a substantial literature in which radiation is not defined locally, but only the total radiation is defined globally, requiring measurements over an entire (possibly very large surface). It might seem sensible to consider that the global radiation is the integral

[3]In the wave theory the direction of a ray is that of the group velocity (see, for example, sec. 2.1 of [5]).

[4]One view of electromagnetic radiation is that it corresponds to any time-dependent electromagnetic field, without reference to flow of energy in that field. I consider this definition to be too broad, and do not consider it further here.

[5]See the Appendix below.

of the local radiation, but this seems not to be done by the advocates of the global definition of radiation, who leave the notion of radiation undefined locally.[6]

A somewhat separate issue is the total amount of radiation emitted by a source at some time. This cannot be determined by observation at a single point, but does require a global network of detectors. Yet, the concept of radiation should be defined independently of whether or not its global integral is measured.

1.2 The Sommerfeld Radiation Condition

When considering mathematical solutions to wave equations that describe radiation, there exist solutions in which energy flows in from "infinity" rather than out towards it. This led Sommerfeld [9, 10] to state his famous "radiation condition" that "the sources must be sources, not sinks of energy. The energy which is radiated from the sources must scatter to infinity; no energy may be radiated from infinity into ... the field."

Sommerfeld's condition seems often interpreted as implying that radiation must be defined at "infinity," where there must be only an outward flow of energy. An extreme, but not entirely uncommon, interpretation is that radiation does not exist except at very large distances. For example, if a source is surrounded by a metallic sphere, some people say that there is no radiation present, because no energy flows to "infinity."

In this author view, Sommerfeld's condition does not define the physical concept of radiation, but is only a mathematical boundary condition. Rather, radiation should be defined locally, and can exist anywhere in space, close to or far from its source.

1.3 Should the Definition of Radiation Be Relativistic?

Since Maxwell's theory of electrodynamics is intrinsically relativistic, concepts of radiation should be also.

The local notion of radiation as having a magnitude and direction indicates that a 3-vector should be part of the definition of radiation. However, it is not possible to associate this 3-vector with a scalar (such as the local energy density) to form a 4-vector. Rather, the 3-vector can be embedded in a Lorentz 4-tensor, although this will not be the focus of the present note.

In the global perspective, the total radiated energy crossing a surface is a scalar, and the total momentum crossing that surface is a 3-vector. For suitable global definitions of these surface integrals, they together form a Lorentz 4-4vector. This also will not be pursued in the present note, but has led various authors to tacit support for Definition B; see, for example, [6, 7, 8].

2 The Poynting Vector

The flow of energy in the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ is described by the Poynting vector [11],

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}, \tag{3}$$

[6] See, for example, [6].

(in Gaussian units) where c is the speed of light in vacuum, so an obvious definition is to identify the Poynting vector with electromagnetic radiation:

A. Electromagnetic radiation is the flow of electromagnetic energy described by the Poynting vector (3).[7,8,9,10]

$$\mathbf{S}_{\rm A} = \mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}. \tag{4}$$

Definition A of radiation encompasses more than electromagnetic waves, since time-independent fields can have a nonzero Poynting vector. For example, in a simple DC circuit energy flows from the battery to the resistor through the intervening space rather than through the wire,[11] as first noted by Poynting [11], and this energy flow is to be called radiation according to Definition A. In the broad sense, this is acceptable usage (and in the quantum view[12] this energy flow involves "virtual" photons[13]).

The electromagnetic field theory of Faraday and Maxwell is a "unified field theory" in which Faraday advocated combining what might be called the "electrostatic" field $-\boldsymbol{\nabla} V$ with what might be called the "electrokinetic" field $-\partial\mathbf{A}/\partial ct$ (where V and $\mathbf{A}$ are the scalar and vector potentials) into the single electric field

$$\mathbf{E} = -\boldsymbol{\nabla} V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\,, \tag{5}$$

and Maxwell identified the wave fields of optics with electromagnetic fields. Definition A of radiation is consistent with the "unified field theory" in that the entire electric and magnetic fields are used to calculate the radiation/Poynting vector. However, attempts to relate

[7] The symbol $\mathbf{S}$ for the Poynting vector appears to come from the German, strahlvektor = radiation vector, as on p. 114 of [12], which implicitly acknowledges Definition A. This definition was using in English literature at least as early as 1927; see p. 5 of [13]. The struggle to define gravitational radiation has led many authors to identify it with a so-called super-Poynting vector [14, 15, 16, 17], which is considered to be a natural extension of the identification of electromagnetic radiation with the Poynting vector (3).

[8] Definition A is more general than Sommerfeld's "radiation condition" [9] that associates radiation only with flow of electromagnetic energy to "infinity" (meaning that the integral of the normal component of the Poynting vector over the surface of any large sphere is nonzero at some times). The "radiation condition" leaves ambiguous the definition of radiation at a finite distances from charges and currents. Indeed, the "radiation condition" has the awkward implication that "radiation" cannot be detected, since only energy that flows to "infinity" is to be called "radiation." It is better to think of the "radiation condition" as a boundary condition in mathematical physics than as a definition of radiation [10].

[9] According to Definition A, radiation is associated with both a charge q with uniform velocity $\mathbf{v}$ and with uniform acceleration $\mathbf{a}$, although in neither case does the charge radiate energy (since there is no "radiation reaction force" $q^2\dot{\mathbf{a}}/2c^2$ in either case). According to the "radiation condition," there is no radiation associated with a uniformly moving charge, while radiation is associated with a uniformly accelerated charge.

[10] A variant is to identify radiation with the Poynting vector (3), but only for time-dependent fields.

[11] See [18] for an instructive model calculation of energy flow in a DC circuit.

[12] The Poynting vector can be calculated for quantum fields. See, for example, sec. 6 of [19].

[13] The spirit of the so-called Weizsäcker-Williams method [20] is to consider the conversion of the "virtual" photons that accompany a uniformly moving charge into "real" ones if that motion is perturbed. In this view, an accelerated electron does not "emit" radiation but rather "sheds" it. *Note, however, that a photon can be "real" (massless) only if it is never detected. Only "virtual" photons are detected, so the popular names "real" and "virtual" have somewhat the inverse of their intended meanings when applied to photons.*

radiation to accelerated charges led to decompositions of the electromagnetic fields into "radiation" and "nonradiation" fields, or into "incident" and "reflected/scattered" fields.[14]

3 Radiation Fields

Considerations of a decomposition of the electric field into "radiation" and "nonradiation" parts likely follows from the calculations by Liènard [24] and by Wiechert [25] of the electromagnetic fields of a single, accelerated charge, which can be summarized as[15]

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_{\rm nonrad} + \mathbf{E}_{\rm rad}, \tag{6}$$

$$\mathbf{E}_{\rm nonrad} = q\left[\frac{\hat{\mathbf{r}} - \mathbf{v}/c}{\gamma^2 r^2(1 - \hat{\mathbf{r}}\cdot\mathbf{v}/c)^3}\right]_{\rm ret}, \qquad \mathbf{E}_{\rm rad} = q\left[\frac{\hat{\mathbf{r}}\times[(\hat{\mathbf{r}} - \mathbf{v}/c)\times\mathbf{a}/c]}{cr(1 - \hat{\mathbf{r}}\cdot\mathbf{v}/c)^3}\right]_{\rm ret}, \tag{7}$$

$$\mathbf{B}(\mathbf{x},t) = [\hat{\mathbf{r}}]_{\rm ret}\times\mathbf{E} = \mathbf{B}_{\rm nonrad} + \mathbf{B}_{\rm rad} = [\hat{\mathbf{r}}]_{\rm ret}\times\mathbf{E}_{\rm nonrad} + [\hat{\mathbf{r}}]_{\rm ret}\times\mathbf{E}_{\rm rad}, \tag{8}$$

where $\mathbf{v} = d\mathbf{x}_q/dt$ is the velocity of the charge, $\mathbf{a} = d^2\mathbf{x}_q/dt^2$ is its acceleration, $\gamma = 1/\sqrt{1 - v^2/c^2}$, the distance from the charge to the observer is $\mathbf{r} = \mathbf{x} - \mathbf{x}_q$, and the retarded time is $t' = t - r/c$.[16] Of course, no measurement can distinguish between $\mathbf{E}_{\rm rad}$ and $\mathbf{E}_{\rm nonrad}$. The decomposition (6) and (8) is purely conceptual, so one must be cautious in assigning physical significance to it (other than that at large distances from a source that contains accelerated charges, the fields fall off inversely with distance).

This decomposition reinforces the notion that "accelerated charges radiate" and that "radiation is due to the acceleration of charges." These views are popularly represented by the "kink model" of radiation, which was perhaps first introduced by Heaviside [28], and more graphically by J.J. Thomson [29].

A decomposition into radiation and nonradiation fields when the sources are charge and

[14] Another decomposition of the fields is due to Helmholtz [21, 22], in which $\mathbf{E} = \mathbf{E}_{\rm irr} + \mathbf{E}_{\rm rot}$ (and $\mathbf{B} = \mathbf{B}_{\rm rot}$) where $\boldsymbol{\nabla}\times\mathbf{E}_{\rm irr} = 0 = \boldsymbol{\nabla}\cdot\mathbf{E}_{\rm rot}$. Then, one can write $\mathbf{S} = \mathbf{S}_{\rm irr} + \mathbf{S}_{\rm rot} = \mathbf{E}_{\rm irr}\times\mathbf{B} + \mathbf{E}_{\rm rot}\times\mathbf{B}$. However, calculation of $\mathbf{E}_{\rm irr}$ and $\mathbf{E}_{\rm rot}$ requires instantaneous knowledge throughout the entire Universe, so the Helmholtz decomposition is of a mathematical rather than physical character. An explanation of radiation that uses the Helmholtz decomposition without awareness of this is [23].

[15] Throughout this note the charges are assumed to be in media with unit relative permittivity and permeability.

[16] An alternative form of eq. (7) was given by Heaviside [26] and later popularized by Feynman [27],

$$\mathbf{E}_{\rm nonrad} = q\left[\frac{\hat{\mathbf{R}}}{R^2}\right] + \frac{q}{c}\left[R\frac{d}{dt}\frac{\hat{\mathbf{R}}}{R^2}\right], \qquad \mathbf{E}_{\rm rad} = \frac{q}{c^2}\left[\frac{d^2\hat{\mathbf{R}}}{dt^2}\right]. \tag{9}$$

Note that Heaviside's v is our c, his $\mu v^2/4\pi = 1$ in Gaussian units, and that his $\mathbf{R}_1$ is our $\hat{\mathbf{R}}$. Then, his eq. (32) can be rewritten as

$$\mathbf{E} = \frac{\mu Q}{4\pi}\left\{\ddot{\mathbf{R}}_1 + \frac{v}{R^2}(R\dot{\mathbf{R}}_1 - 2\mathbf{R}_1\dot{R} + v\mathbf{R}_1)\right\} \to Q\left[\frac{\hat{\mathbf{R}}}{R^2} + \frac{R}{c}\frac{d}{dt}\frac{\hat{\mathbf{R}}}{R^2} + \frac{1}{c^2}\frac{d^2\hat{\mathbf{R}}}{dt^2}\right]_{\rm ret}, \tag{10}$$

which is Feynman's expression (9).

current densities ρ and $\mathbf{J}$ can be made using[17]

$$\mathbf{E}(\mathbf{x},t) = \int \frac{[\rho]\,\hat{\mathbf{R}}}{R^2}\,d^3\mathbf{x}' + \frac{1}{c}\int \frac{([\mathbf{J}]\cdot\hat{\mathbf{R}})\hat{\mathbf{R}} + ([\mathbf{J}]\times\hat{\mathbf{R}})\times\hat{\mathbf{R}}}{R^2}\,d^3\mathbf{x}' + \frac{1}{c^2}\int \frac{([\dot{\mathbf{J}}]\times\hat{\mathbf{R}})\times\hat{\mathbf{R}}}{R}\,d^3\mathbf{x}', \tag{13}$$

$$\mathbf{B}(\mathbf{x},t) = \frac{1}{c}\int \frac{[\mathbf{J}]\times\hat{\mathbf{R}}}{R^2}\,d^3\mathbf{x}' + \frac{1}{c^2}\int \frac{[\dot{\mathbf{J}}]\times\hat{\mathbf{R}}}{R}\,d^3\mathbf{x}', \tag{14}$$

where $\mathbf{R} = \mathbf{x} - \mathbf{x}'$ and $[\mathbf{J}] = \mathbf{J}(\mathbf{x}', t' = t - R/c)$. The radiation fields are the final terms in eqs. (13)-(14),

$$\mathbf{E}_{\rm rad}(\mathbf{x},t) = \frac{1}{c^2}\int \frac{([\dot{\mathbf{J}}]\times\hat{\mathbf{R}})\times\hat{\mathbf{R}}}{R}\,d^3\mathbf{x}', \qquad \mathbf{B}_{\rm rad}(\mathbf{x},t) = \frac{1}{c^2}\int \frac{[\dot{\mathbf{J}}]\times\hat{\mathbf{R}}}{R}\,d^3\mathbf{x}'. \tag{15}$$

The decomposition into radiation and nonradiation fields leads many people to a different definition of radiation:

B. Radiation is the flow of energy described by the part of the Poynting vector due to the radiation fields,

$$\mathbf{S}_B = \mathbf{S}_{\rm rad} = \frac{c}{4\pi}\mathbf{E}_{\rm rad}\times\mathbf{B}_{\rm rad}. \tag{16}$$

However, neither the vectors $\mathbf{S}$ nor $\mathbf{S}_{\rm rad}$ can be written as a single volume integral over the source charges and currents,

$$\mathbf{S},\ \mathbf{S}_{\rm rad} \neq \int \frac{\mathbf{s}([\rho]\,,[\mathbf{J}]\,,[\dot{\mathbf{J}}],\hat{\mathbf{R}})}{R^2}\,d^3\mathbf{x}', \tag{17}$$

where $\mathbf{s}$ is some vector function (possibly including spatial derivatives of $[\rho]$ and $[\mathbf{J}]$).[18] Rather, we note that Poynting's theorem [11] can be written in the form

$$\boldsymbol{\nabla}\cdot\mathbf{S} = -\frac{\partial u}{\partial t} - \mathbf{J}\cdot\mathbf{E}, \tag{19}$$

[17]Equations (13)-(14) first appear in [30], although versions of their Fourier transforms appear in [31, 32], and more explicitly in [33, 34]. An alternative approach is to integrate the wave equations,

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi\boldsymbol{\nabla}\rho + \frac{4\pi}{c}\frac{\partial\mathbf{J}}{\partial t} \qquad \text{and} \qquad \nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = -\frac{4\pi}{c}\boldsymbol{\nabla}\times\mathbf{J}, \tag{11}$$

using the method of Lorenz [35] to obtain

$$\mathbf{E} = -\int \frac{[\boldsymbol{\nabla}'\rho]}{R}\,d^3\mathbf{x}' - \frac{1}{c^2}\int \frac{[\dot{\mathbf{J}}]}{R}\,d^3\mathbf{x}' \qquad \text{and} \qquad \mathbf{B} = \frac{1}{c}\int \frac{[\boldsymbol{\nabla}'\times\mathbf{J}]}{R}\,d^3\mathbf{x}'. \tag{12}$$

With effort, the derivatives $\boldsymbol{\nabla}'$ can to transformed away to yield eqs. (13)-(14).

[18]One way to see this is to take the d'Alembertian of the Poynting vector (3),

$$\begin{aligned}\frac{4\pi}{c}\left(\nabla^2\mathbf{S} - \frac{1}{c^2}\frac{\partial^2\mathbf{S}}{\partial t^2}\right) &= \left(\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}\right)\times\mathbf{B} + \mathbf{E}\times\left(\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}\right)\\ &\quad +2\sum_{w=x,y,z}\frac{\partial\mathbf{E}}{\partial w}\times\frac{\partial\mathbf{B}}{\partial w} - \frac{2}{c^2}\frac{\partial\mathbf{E}}{\partial t}\times\frac{\partial\mathbf{B}}{\partial t}\,.\end{aligned} \tag{18}$$

Since the righthand side of eq. (18) cannot be expressed directly in terms of charges and currents (without becoming an integral equation by use of eqs. (11)-(12)), no solution of form (17) exists.

where $u = (E^2 + B^2)/8\pi$ is the density of energy in the electromagnetic field. This has the implication that both a time-varying field-energy density u and the electric current $\mathbf{J}$ act as sources for the Poynting vector. See [36, 37, 38, 39, 40] for analytic discussions of this in the case of pulsed, point (Hertzian) dipoles.

Thus, although we can identify "radiation fields" which depend only on accelerated charges, we cannot say electromagnetic radiation is the flow of electromagnetic energy corresponding to some or all of the Poynting vector (3) such that this radiation depends only on charges and currents in an integration over (retarded) source terms. It appears to this author that the decomposition of the electromagnetic fields into "radiation" and "nonradiation" parts (which goes against the Maxwellian vision of a unified field theory) does not accomplish its underlying goal of relating "radiation" to accelerated charges alone.[19] As such, it is preferable to use Definition A for radiation as being described by the entire Poynting vector.

Furthermore, local measurement of the electromagnetic fields cannot separate them into "radiation" and "nonradiation" components. This can only be done with knowledge of the distance source current density $\mathbf{J}$. Hence, Definition B of radiation is not a local definition, and should be disfavored for this reason alone.

4 Radiation of Angular Momentum

The angular momentum of an electromagnetic field is defined as[20]

$$\mathbf{L}_{\text{field}} = \int \mathbf{r} \times \frac{\mathbf{E} \times \mathbf{B}}{4\pi c}\, d\text{Vol}. \tag{20}$$

According to definition A,eq. (20) also described the radiated angular momentum, but if If we adopt definition B for radiation, we would then say that the radiated angular momentum is

$$\mathbf{L}_{\text{rad}} = \int \mathbf{r} \times \frac{\mathbf{E}_{\text{rad}} \times \mathbf{B}_{\text{rad}}}{4\pi c}\, d\text{Vol} \qquad \text{(Definition B)}. \tag{21}$$

However, for a source centered on the origin, the radiation fields $\mathbf{E}_{\text{rad}}$ and $\mathbf{B}_{\text{rad}}$ are transverse to $\mathbf{r}$ at large distances, such that the integrand of eq. (21) vanishes at large distances, and we would be led to say that no angular momentum can be radiated to "infinity."[21]

Hence, Definition A is to be preferred over Definition B.

[19] An example of an extended system in which Definitions A and B lead to the same notion of "radiation" is a uniform sheet of charge that is given a constant velocity in its plane at $t = 0$. See [41] for computation of the radiation here according to Definition B.

[20] See, for example, p. 608 of [42] and eq. (A.9) of [43].

[21] As discussed in prob. 5 of [44], the radiation of angular momentum in the far zone of an elliptically polarized Hertzian electric dipole is associated with the cross product of the part of the electric field that varies as $1/r$ with the part of the magnetic field that varies as $1/r^2$, and conversely. Hence, the parts of the fields that vary as $1/r^2$ should be considered as contributing to "radiation," as in Definition A.

5 The Issue of Cause and Effect

The equations (6)-(8) and (13)-(14) express the electromagnetic fields in terms of charges and currents. This can give the impression that Maxwell's equations imply that fields are "caused" by charges and currents. However, most electrical currents are "caused" by electric fields, and many electric charge distributions are "induced" by electric fields. That is, Maxwell's electrodynamics is a complete logical system only when his four differential equations for the fields are supplemented with the laws,

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right), \qquad \mathbf{f} = \rho\mathbf{E} + \frac{\mathbf{J}}{c} \times \mathbf{B}, \tag{22}$$

for the force $\mathbf{F}$ on an individual charge (Lorentz) and for the force density $\mathbf{f}$ on charge and current densities, respectively.[22] Thus, categorical identification of "cause" and "effect" (or "before" and "after") in electrodynamics is not possible in general, being dependent on an assumption as to what constitutes the initial conditions.

Definition B of radiation tends to be associated with a view that the relevant initial conditions involve knowledge of the charge and current distributions, whereas Definition A of radiation is more neutral as to the assumption as to the initial conditions.

An even more explicit attempt to associate the concept of radiation with "before" and "after" is considered in sec. 6.

6 Time-Harmonic Fields

Many important examples of the flow of electromagnetic energy involve such a narrow range of frequencies that the approximation of a single (angular) frequency ω is sufficient. In this approximation the time average of any quantity (at a given point in space) is constant. In particular, the time-average field energy density $\langle u \rangle$ is constant, such that the time-average of Poynting's theorem (19) reads

$$\boldsymbol{\nabla} \cdot \langle \mathbf{S} \rangle = -\langle \mathbf{J} \cdot \mathbf{E} \rangle, \tag{23}$$

which permits the interpretation that the time-average Poynting vector has no sources in current-free regions. This contrasts with the case of the Poynting vector for fields with arbitrary time dependence, for which a changing field energy density $\partial u/\partial t$ acts as a source.

The present note concerns the definition of radiation for a collection of charges, particularly those in conductors, in which case the velocities are extremely low and the lab frame is essentially the instantaneous rest frame of all of the charges. Then, the motion of each charge is well approximated as that of an ideal, oscillating, point (Hertzian) electric dipole [46] with (complex) moment $\mathbf{p}$, for which the electromagnetic fields are (see, for example, sec. 9.2 of [42])

$$\mathbf{E} = k^2 p(\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \times \hat{\mathbf{r}}\, \frac{e^{i(kr-\omega t)}}{r} + p[3(\hat{\mathbf{p}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{p}}] \left(\frac{1}{r^3} - \frac{ik}{r^2}\right) e^{i(kr-\omega t)}, \tag{24}$$

[22] The Lorentz force density in eq. (22) is not reliable for force computations in some cases involving macroscopic, permeable media. See, for example, [45].

$$\mathbf{B} = k^2 p (\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \left(\frac{1}{r} - \frac{1}{ikr^2} \right) e^{i(kr-\omega t)}. \tag{25}$$

The "radiation fields" of a Hertzian dipole are

$$\mathbf{E}_{\rm rad} = k^2 p (\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \times \hat{\mathbf{r}} \, \frac{e^{i(kr-\omega t)}}{r}, \qquad \mathbf{B}_{\rm rad} = k^2 p (\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \frac{e^{i(kr-\omega t)}}{r}. \tag{26}$$

Hence, Definitions A and B for the time-average radiation of a collection of oscillating charges q_j are,

A. The time-average radiation is defined to be the (time-average) Poynting vector of the total fields of the charges.

$$\langle \mathbf{S}_A \rangle = \langle \mathbf{S} \rangle = \frac{c}{8\pi} Re \left(\sum_j \mathbf{E}(q_j) \times \sum_k \mathbf{B}^\star(q_k) \right). \tag{27}$$

B. The time-average radiation is defined to be the (time-average) Poynting vector due only to the "radiation fields" of the charges,[23]

$$\langle \mathbf{S}_B \rangle = \frac{c}{8\pi} Re \left(\sum_j \mathbf{E}_{\rm rad}(q_j) \times \sum_k \mathbf{B}^\star_{\rm rad}(q_k) \right). \tag{28}$$

For a single oscillating charge we find that

$$\langle \mathbf{S} \rangle = \langle \mathbf{S}_A \rangle = \langle \mathbf{S}_B \rangle = k^4 \left| p \right|^2 \frac{\left| \hat{\mathbf{r}} \times \hat{\mathbf{p}} \right|^2}{r^2} \hat{\mathbf{r}}, \tag{29}$$

where unit vector $\hat{\mathbf{r}}$ points from the average position of the charge to the observer.

To see that the quantities $\langle \mathbf{S}_A \rangle$ and $\langle \mathbf{S}_B \rangle$ are not the same when more that one charge is accelerated it suffices to consider a system of only two charges, q_1 and q_2, at (average) positions $\mathbf{x}_1$ and $\mathbf{x}_2$, with (complex) oscillating dipole moments $\mathbf{p}_1$ and $\mathbf{p}_2$. Then,

$$\begin{aligned} \mathbf{E}(\mathbf{x}, t) \;=\; & k^2 p_1 (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \times \hat{\mathbf{r}}_1 \frac{e^{i(kr_1-\omega t)}}{r_1} + p_1 [3(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{r}}_1)\hat{\mathbf{r}}_1 - \hat{\mathbf{p}}_1] \left(\frac{1}{r_1^3} - \frac{ik}{r_1^2} \right) e^{i(kr_1-\omega t)} \\ & + k^2 p_1 (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) \times \hat{\mathbf{r}}_2 \frac{e^{i(kr_2-\omega t)}}{r_2} + p_2 [3(\hat{\mathbf{p}}_2 \cdot \hat{\mathbf{r}}_2)\hat{\mathbf{r}}_2 - \hat{\mathbf{p}}_2] \left(\frac{1}{r_2^3} - \frac{ik}{r_2^2} \right) e^{i(kr_2-\omega t)}, \end{aligned} \tag{30}$$

$$\mathbf{B}(\mathbf{x}, t) = k^2 p_1 (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \left(\frac{1}{r_1} - \frac{1}{ikr_1^2} \right) e^{i(kr_1-\omega t)} + k^2 p_2 (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) \left(\frac{1}{r_2} - \frac{1}{ikr_2^2} \right) e^{i(kr_2-\omega t)}, \tag{31}$$

[23] It is always possible to represent a time-varying electromagnetic field as a sum of electromagnetic plane waves, of which some are propagating (homogeneous, of form $\mathbf{E}\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, $\mathbf{B} = \hat{\mathbf{k}} \times \mathbf{E}$, where the constant fields $\mathbf{E}$ and $\mathbf{B}$ obey $\mathbf{E} \cdot \mathbf{k} = 0 = \mathbf{B} \cdot \mathbf{k}$) and some are evanescent (inhomogeneous) [47]. We might then entertain Definition C, that "radiation" is $\langle \mathbf{S}_C \rangle = (c/8\pi) Re \sum \mathbf{E}_m \times \mathbf{B}^\star_n \, e^{i[(\mathbf{k}_m - \mathbf{k}_n)\cdot\mathbf{r} - (\omega_m - \omega_n)t]}$, *i.e.*, the (time-average) Poynting vector formed only from the propagating electromagnetic fields. I believe that Definition C is equivalent to Definition B, in that only the "radiation fields" propagate far from their sources and are therefore represented by the propagating waves in Definition C. Then, the objection to Definition B discussed below also applies to Definition C.

where $\mathbf{r}_j = \mathbf{x} - \mathbf{x}_j$, and

$$
\begin{aligned}
\frac{8\pi}{c} \langle \mathbf{S}_A \rangle \;=\;& k^4 \left|p_1\right|^2 \frac{\left|\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1\right|^2}{r_1^2} \hat{\mathbf{r}}_1 + k^4 \left|p_2\right|^2 \frac{\left|\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2\right|^2}{r_2^2} \hat{\mathbf{r}}_2 \\
& + k^4 [(\hat{\mathbf{r}}_1 + \hat{\mathbf{r}}_2)(\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \cdot (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) - (\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_1)(\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \\
& \quad - (\hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_2)(\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2)] \left\{ \frac{2 Re[p_1 p_2^\star \, e^{ik(r_1 - r_2)}]}{r_1 r_2} \right. \\
& \qquad \left. + Re \left[\frac{p_1 p_2^\star \, e^{ik(r_1 - r_2)}}{ikr_1 r_2^2} + \frac{p_2 p_1^\star \, e^{ik(r_2 - r_1)}}{ikr_2 r_1^2} \right] \right\} \qquad (32) \\
& + k^2 [3(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{r}}_1)\hat{\mathbf{r}}_1 - \hat{\mathbf{p}}_1] \times (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) Re \left[p_1 p_2^\star \, e^{ik(r_1 - r_2)} \left(\frac{1}{r_1^3} - \frac{ik}{r_1^2} \right) \left(\frac{1}{r_2} + \frac{1}{ikr_2^2} \right) \right] \\
& + k^2 [3(\hat{\mathbf{p}}_2 \cdot \hat{\mathbf{r}}_2)\hat{\mathbf{r}}_2 - \hat{\mathbf{p}}_2] \times (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) Re \left[p_2 p_1^\star \, e^{ik(r_2 - r_1)} \left(\frac{1}{r_2^3} - \frac{ik}{r_2^2} \right) \left(\frac{1}{r_1} + \frac{1}{ikr_1^2} \right) \right].
\end{aligned}
$$

Then, $(8\pi/c) \langle \mathbf{S}_B \rangle$ equals only the first three lines of eq. (32).

Thus, $\langle \mathbf{S}_A \rangle = \langle \mathbf{S}_B \rangle$ in the "far zone" (where $kr_j \gg 1$ for all charges q_j), but they differ in the "near zone" close to the source charges.

The total time-average energy density $\langle u \rangle$ in the electromagnetic fields of the oscillating charges is constant in time at every point in space outside the charges themselves, so energy conservation (Poynting's theorem) implies that

$$
\boldsymbol{\nabla} \cdot \langle \mathbf{S}_A \rangle = -\frac{\partial \langle u \rangle}{\partial t} = 0. \qquad (33)
$$

However,

$$
\begin{aligned}
\frac{4\pi}{ck^4} \boldsymbol{\nabla} \cdot \langle \mathbf{S}_B \rangle \;=\;& k \frac{Im[p_1 p_2^\star \, e^{ik(r_1 - r_2)}]}{r_1 r_2} [(\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_2)^2 - (\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_1)^2] \\
& - \frac{Re[p_1 p_2^\star \, e^{ik(r_1 - r_2)}]}{r_1 r_2} \left[\frac{(\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_2)^2}{r_1} + \frac{(\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_1)^2}{r_2} \right. \\
& \quad + (\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2 - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \cdot (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) \\
& \quad - \left(\frac{1}{r_1} + \frac{1}{r_2} \right) (\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_1)(\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_2) \\
& \quad - (\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2) \cdot \left(\frac{(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{p}}_1)\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2}{r_1} + \frac{(\hat{\mathbf{r}}_2 \cdot \hat{\mathbf{p}}_2)\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1}{r_2} \right) \\
& \quad + (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_2) \cdot (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_2) \left(\frac{\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2}{r_1} - \frac{1}{r_2} \right) \\
& \quad \left. + (\hat{\mathbf{r}}_1 \times \hat{\mathbf{p}}_1) \cdot (\hat{\mathbf{r}}_2 \times \hat{\mathbf{p}}_1) \left(\frac{\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2}{r_2} - \frac{1}{r_1} \right) \right], \qquad (34)
\end{aligned}
$$

which is nonzero in the "near zone" (and outside the charges).[24] This means that there are sources (and sinks) of the Poynting flux $\langle \mathbf{S}_B \rangle$ in the "near zone" other than the charges

[24]As expected, eq. (34) vanishes when $\mathbf{r}_1 = \mathbf{r_2}$ even if $\mathbf{p}_1 \neq \mathbf{p}_2$, and when either p_1 or p_2 is zero.

themselves. That is, the other part of the time-average Poynting vector, $\langle \mathbf{S} \rangle - \langle \mathbf{S}_B \rangle$, delivers energy steadily to some current-free regions in the "near zone," where that energy is (mathematically) "converted" into the flow $\langle \mathbf{S}_B \rangle$ and transported to other regions of the "near zone," where that energy is "reconverted" into the flow $\langle \mathbf{S} \rangle - \langle \mathbf{S}_B \rangle$.[25,26]

In contrast, the time-average flow of energy described by $\langle \mathbf{S} \rangle = \langle \mathbf{S}_A \rangle$ moves smoothly from source currents through the "near zone" and on to the "far zone" (or into sink currents in the "near zone").

7 Incident and Reflected/Scattered Radiation

Another notion about radiation that comes from optics is that it can usefully be decomposed into "incident" and "reflected" (or "scattered") radiation.[27] This decomposition is based on three assumptions that are not generally valid: all charges and currents reside in two widely separated regions, such that the back reaction of the "scattered" waves on the source region can be neglected; the observer is many wavelengths away from and charges and currents; and that the directions of the "incident" and "scattered" waves at the observer are obvious, such that a directional detector can distinguish between the "incident" and the "scattered" radiation.

In the "near zone" of charges and currents, these assumptions are not realistic, and the decomposition does not give very meaningful results there. To see this in more detail, consider the decomposition

$$\mathbf{E} = \mathbf{E}_{\text{in}} + \mathbf{E}_{\text{scat}}, \qquad \mathbf{B} = \mathbf{B}_{\text{in}} + \mathbf{B}_{\text{scat}}. \tag{35}$$

Then the Poynting vector can be written

$$\begin{aligned} \mathbf{S} &= \frac{c}{4\pi}(\mathbf{E}_{\text{in}} + \mathbf{E}_{\text{scat}} \times (\mathbf{B}_{\text{in}} + \mathbf{B}_{\text{scat}}) \\ &= \frac{c}{4\pi}\mathbf{E}_{\text{in}} \times \mathbf{B}_{\text{in}} + \frac{c}{4\pi}\mathbf{E}_{\text{scat}} \times \mathbf{B}_{\text{scat}} + \frac{c}{4\pi}\mathbf{E}_{\text{in}} \times \mathbf{B}_{\text{scat}} + \frac{c}{4\pi}\mathbf{E}_{\text{scat}} \times \mathbf{B}_{\text{in}} \\ &= \mathbf{S}_{\text{in}} + \mathbf{S}_{\text{scat}} + \mathbf{S}_{\text{other}}, \end{aligned} \tag{36}$$

where

$$\mathbf{S}_{\text{in}} = \frac{c}{4\pi}\mathbf{E}_{\text{in}} \times \mathbf{B}_{\text{in}}, \tag{37}$$

[25] One of the few discussions that shows awareness of this complicated scenario is given in [37]. See also [39, 40].

[26] Another awkwardness of Definition B is that in the "radiation" energy density $\langle u_{\text{rad}} \rangle = \langle u_{B,\text{rad}} \rangle + \langle u_{E,\text{rad}} \rangle$ the magnetic and electric "radiation" energy densities are not, in general, equal to one another (although they are everywhere equal for idealized Hertzian dipole radiators). Lack of awareness of this fact has led to numerous faulty analyses of the relation of circuit reactance to electromagnetic fields, as reviewed in [48, 49].

[27] In thermal physics one speaks of energy "radiated" and "absorbed" by a material surface at some temperature. If that surface is in thermal equilibrium with its surroundings, the rates of "radiation" and "absorption" of energy are equal. The "radiated" and "absorbed" energy is in the form of incoherent electromagnetic waves emitted by individual atoms or molecules in the material of the surface. Hence, this form of "radiation" is not subject to the issues addressed in this note, which concern coherent "radiation" by a system of charges and currents.

$$\mathbf{S}_{\rm scat} = \frac{c}{4\pi}\mathbf{E}_{\rm scat} \times \mathbf{B}_{\rm scat}, \tag{38}$$

$$\mathbf{S}_{\rm other} = \frac{c}{4\pi}\mathbf{E}_{\rm in} \times \mathbf{B}_{\rm scat} + \frac{c}{4\pi}\mathbf{E}_{\rm scat} \times \mathbf{B}_{\rm in}. \tag{39}$$

The existence of the nontrivial cross term $\mathbf{S}_{\rm other}$ does not permit a decomposition of the energy flow into only "incident" and "scattered" terms. In general, the energy "scattered" from a point is not only due to the direct effect of the "incident" wave, but also due to energy that arrives at that point as the result of "scattering" of the "incident" energy off other charges or currents in the "near zone."

Of course, the decomposition (35) goes against the spirit of the "unified field theory" of classical electromagnetism, so it is to be expected that it leads to unsatisfactory results in general.[28]

8 Radiation Near Good Conductors

In case of good conductors with simple geometry, such as planes, the decomposition (35) of the fields can often be accomplished by solving a boundary-value problem. An interesting example of this is the decomposition of the fields (whose phase velocity exceeds c) inside a rectangular waveguide into plane waves that propagate with velocity c and "zig-zag" down the guide [51]. It is still best to use the total Poynting vector to describe the flow of energy, which is parallel to the surface of the conductors in the region just outside them. On the scale of a wavelength away from the surface the flow can become complex, as, for example, in a waveguide [52] or in the "whirlpools" of energy flow that occur when a Gaussian laser beam reflects off a good conductor [53].

Some people appear reluctant to accept Definition A because it implies that accelerated charges in good/perfect conductors do not radiate. As noted at least as early as 1897 in a discussion of radiation by wires [54], the tangential component of the electric field must vanish at the surface of a good/perfect conductor.[29] As a consequence the total Poynting vector has no component perpendicular to the surface of a good/perfect conductor at any time [58], and hence there is no net flow of energy into or out of a good conductor.[30] If we identify radiation with the total flow of energy, as in Definition A, then we arrive at the so-called "radiation paradox" that the good conductors of antennas do not radiate (see sec. 6 of [60]). Rather, the radiation originates in the power source (which must contain some elements in which charges flow in other than good conductors), and is thereafter guided

[28]For a discussion of the surprising character of $\mathbf{S}_{\rm scat}$ for a plane wave incident on a small conducting sphere, see [50].

[29]This permits formulation of an integral equation for the currents in a good/perfect conductor in terms of a time-harmonic "source" voltage. After solving numerically for the currents (see, for example, [55]), the fields (and the Poynting vector/radiation) can then be calculated. For an analytic review of this approach, see [56]. In the case of general time dependence, Maxwell's equations and the equations of motion of charges/currents can be integrated numerically for time steps on a mesh, in which the good/perfect conductor boundary condition is enforced at each step [57].

[30]In a microscopic model of currents as moving charges, there is a small, time-dependent kinetic energy associated with the currents, which energy is exchanged with the energy of the electromagnetic fields outside the conductor. This is accounted for by consideration of the small imaginary part of the conductivity in good, but not perfect conductors. See sec. 3.1 of [59].

by the good conductors of the transmission line (if any) and of the nominal antenna. In this view, the nominal antenna plays only a somewhat passive role, whereas many antenna enthusiasts prefer a vision in which the nominal antenna plays a more active role, and often favor Definition B.

9 Summary

In the view of this author, Definition A of electromagnetic radiation as being the flow of electromagnetic energy described by the total Poynting vector is preferred over Definition B that radiation is only that part of the Poynting vector due to the "radiation fields."

1. Definition B is based on a decomposition of the electric field into terms that cannot separately be measured, and thereby violates the spirit of the unified field theory of Faraday and Maxwell. Definition B can only be implement via knowledge of the source currents, rather than fields measured by an observer. Definition A is local, which Definition B is not.

2. Definition B tends to become associated with the vision that "currents cause fields/ radiation," while omitting to acknowledge that "fields/radiation cause currents;" whereas Definition A defines radiation in terms of fields with less underlying implication of "cause" and "effect."

3. Definition B does not lead to a concept of "radiation" as being due only to accelerated charges (although the related concept of "radiation fields" does associate parts of the electric and magnetic fields with accelerated charges).

4. In the case of time-harmonic currents and fields, Definition A, but not Definition B, is such that the time-average radiation (flow of energy) can be traced only to currents.

5. Definition B implies that angular momentum could not be radiated to large distances from a source, while Definition A is consistent with such radiation.

6. Definition A is more compatible than Definition B with the semiclassical view of radiation considered in the Weizsäcker-Williams model [20].

The appealing concept of "radiation fields" has clear physical significance only in the "far zone," where other field components can generally be neglected,[31] and where Definitions A and B of radiation are effectively the same.[32] While "radiation fields" can also be defined in

[31]Characterization of the angular momentum of electromagnetic fields in the "far zone" requires consideration of terms that fall off as $1/r^2$ as well as the dominant terms that fall off as $1/r$.

[32]Teitelboim [7] has developed a Lorentz-invariant partition of the field energy-momentum 4-tensor of a single electric charge into pieces he calls "bound" and "radiated". In this view, a uniformly accelerated charge is a sink of bound energy-momentum and a source of radiated energy-momentum, with the fluxes of these two being equal and opposite close to the charge. See also sec. 3, paper II and sec. 3, paper III of [8]. Teitelboim's "radiation" corresponds to definition B, and hence our present arguments against the general utility of definition B indicate that Teitelboim's "split" of the energy-momentum tensor cannot be extended to the case of two or more charges.

the "near zone," use of only these fields to characterize the flow of electromagnetic energy in this region is, to this author, unsatisfactory.[33] In contrast, identifying the total Poynting vector with radiation provides a simple, consistent description of the flow of electromagnetic energy in all regions, while clarifying that in general not all accelerated charges emit radiation and that not all radiation is due to accelerated charges.

Similarly, a decomposition of the electromagnetic fields into "incident" and "scattered" terms does not lead to a satisfactory description of the flow of energy close to charges and currents, although the results of this decomposition are very appealing in the "far zone."

The decompositions of the electromagnetic fields into "radiation" and "nonradiation" terms, or into "incident" and "scattered" terms have the possible merit of implying that all accelerated charges are directly involved in the process of "radiation", while the total Poynting vector traces "radiation" back to only those accelerated charges not in good/perfect conductors, and relegates charges in the latter to the supporting role of "guiding" rather than generating the "radiation."[34]

Although the classical view of radiation as energy flow according to the total Poynting vector (Definition A) does not fully capture the subtlety of the quantum view of the behavior of photons in the "near zone" of matter (as discussed briefly at the end of sec. 1), it remains the most consistent classical interpretation of radiation. The total Poynting vector provides a detailed (perhaps overly detailed) description of the flow of energy/radiation with minimal bias as to situational notions of "cause" and "effect."

A Appendix: The Classical Path of Radiation[35]

If one accepts definition A, that (classical) electromagnetic is described by the Poynting vector (1), then radiation has a classical path, namely lines of the Poynting vector.

Such lines have been computed for a version of Young's double-slit experiment in [63], as shown on the next page. The lines pass through one slit or the other, and do not cross.

In the (standard) quantum view, photons do not have a classical path and cannot be said to pass through one slit or the other (if the interference pattern is observed). Lines of the Poynting vector correspond to lines of the probability current,[36] and so describe the statistical behavior of a photon (as could be observed during many repetitions of a double slit experiment with single photons), rather than the precise path of a single photon.[37]

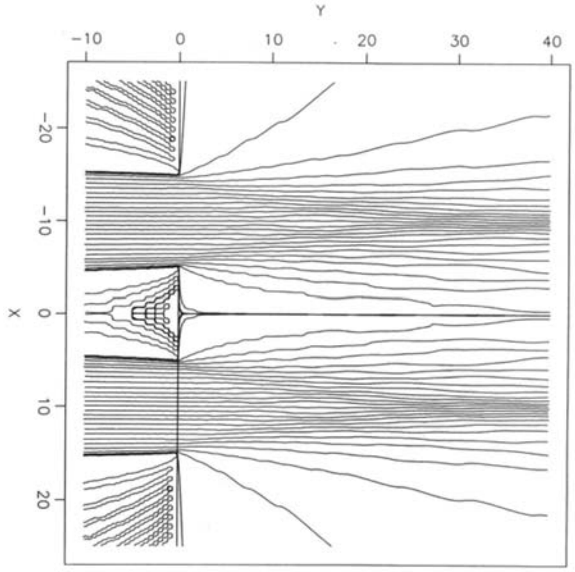
[33]Some authors who appear to favor Definition B (see, for example [61, 62]) seem to this author to be trying to find a particle-like description of classical radiation by emphasizing "time domain analysis" of narrow pulses. However, a "time-domain analysis" of electromagnetic fields is still a field theory, in which the flow of energy obeys Poynting's theorem (19) and a changing density of electromagnetic field energy acts as a source of further energy flow (or a sink of incoming flow). See, for example, [36, 37, 39, 40]. The classical field theory of Maxwell does not contain the particle-like quantum notion that a photon cannot "split" into two photons (and that two photons cannot merge into one).

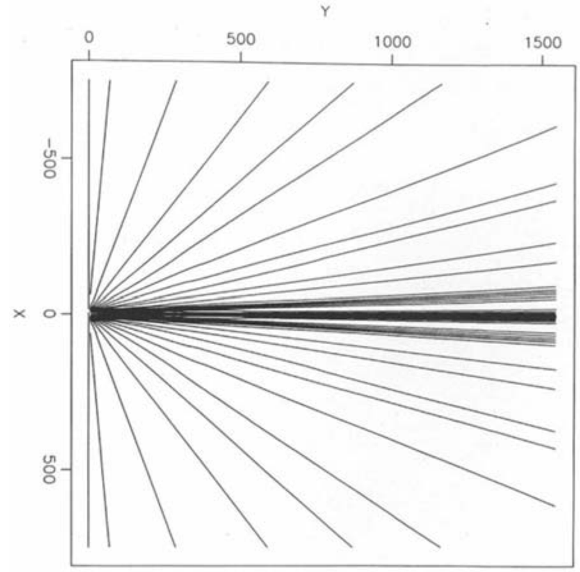
[34]While no net energy flows across the surface of a good/perfect conductor, momentum does. This momentum flow, which can be described by Maxwell's stress/momentum tensor, provides in principle a classical view of how charges in good conductors affect/guide Poynting flux/radiation.

[35]This section was written in May 2019.

[36]See, for example, eq. (40) of [64].

[37]In the view of Bohm [66], individual photons do have a "hidden" pointlike character, in which the "hidden" point follows a line of the Poynting vector/probability current.





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A double-slit experiment with electrons rather than photons is discusses in the Bohmian view in [67], where Fig. 3 is very similar to that of [63] shown above.

A lengthy discussion of photon wave functions in the Bohmian view is given in [68].

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