Does Centrifugal Force Affect Electric Currents?

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1 Problem

Taking the answer to be YES, consider a long cylindrical conductor of radius a that rotates with (small) angular velocity ω in negligible external electromagnetic fields. What is the surface electric charge density, and the electric potential difference between the axis and the surface of the cylinder, in the steady state?[1]

You may assume that the expansion of the cylinder due to its rotation is negligible.[2]

2 Solution

We suppose that the conductor when at rest in zero external fields has number density n_0 of conduction electrons. When the conductor is rotating we suppose that the electric charge density of the cylinder can be described as $\rho_+ + \rho_-$ (plus addition charges with zero average charge density), where $\rho_+ = en_0$, and $-e$ is the charge of an electron. If the cylinder were not rotating, we would have $\rho_- = -en_0$. For a cylinder rotating with angular velocity ω about its axis, the centrifugal force on a conduction electron of mass m at distance r from the axis of rotation is $F_r = m\omega^2 r$. The conduction electrons are pushed outwards, such that their charge density ρ_- is smaller in magnitude than ρ_+, and a negative surface charge density σ accumulates on the surface $r = a$.

In the steady state, there must exist a radial electric field E_r such that its radial force $F_E = -eE_r$ on a conduction electron is equal and opposite to the centrifugal force F_r, so that the total force on the electron is zero.[3]

The radial electric field follows from the Maxwell equation $\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi\rho$ (in Gaussian units) as,

$$E_r = 2r(\rho_+ + \rho_-) = -\frac{F_E}{e} = \frac{F_r}{e} = \frac{m\omega^2 r}{e}\,. \qquad (1)$$

assuming that ρ_- is a constant. Hence,

$$\rho_- = \frac{m\omega^2}{2e} - \rho_+ = \frac{m\omega^2}{2e} - en_0. \qquad (2)$$

[1]This example exhibits (very small) unipolar induction, which is a correction to that found in the more usual cases of a rotating conductor in an external magnetic field (studied by Faraday in Arts. 84-100 of [1]), and a rotating magnetized cylinder (studied by Faraday in Arts. 3084-3122 of [2]; see also[3]).

[2]See [4] for discussion of the case of high angular velocity such that the rotating cylinder might pull itself apart.

[3]In principle, the rotating charge distribution creates an axial magnetic field that exerts a radial force of order v^2/c^2, where c is the speed of light in vacuum, on the conduction electrons whose azimuthal velocity is $v = \omega r$. For small angular velocity ω we neglect this force.

The electric potential difference between the axis of the cylinder and its surface is,

$$\Delta V = \int_0^a E_r \, dr = \frac{m\omega^2 a^2}{2e} \, . \tag{3}$$

If the rotating cylinder is isolated and remains electrically neutral, then its surface supports an electric charge density σ given by,

$$\sigma = -\frac{(\rho_+ + \rho_-)\text{Volume}}{\text{Area}} = -\frac{m\omega^2}{2e}\frac{a}{2} \, . \tag{4}$$

3 Comments

While the preceding analysis seems elementary, it would not have been endorsed by Maxwell, who did not consider that electric charge was carried by particles with a fixed ratio m/e of mass to charge.

Only after J.J. Thomson (1897) [5] measured that ratio for "cathode rays" (electrons) did the notion of (elementary) charged particles become generally accepted.[4] Prior to this, the influence of Faraday remained strong, that while electric charge was transported during the process of electrolysis, where *for this case of electro-chemical decomposition...the* chemical power*...is in direct proportion to the absolute quantity of electricity which passes*,[5] neither Faraday nor Maxwell would identify chemical power with mass (as is now the accepted interpretation).[6]

Some of Maxwell's comments on this theme appear in Arts. 568-577 of [10] (1873). Faraday's studies of electrolysis are mentioned in Art. 569, and the issue of whether magnetic energy includes mechanical kinetic energy was addressed in Arts. 574-577, including reports of experiments with null results (see also [11]).

Helmholtz (1881) [12] came close to associating electric currents with charge carriers of definite mass, but still considered this as speculation.

Lodge (1876) followed Maxwell with experiments searching for momentum transfer during electrolysis, but also with null results, sec. 16 of [13], and Art. 189 of [14]. Hertz (1881) [15] also reported a null experimental result. A positive result was reported by Colley (1882) [16], that when a vial of electrolyte was dropped, a brief electric potential difference between its top and bottom ends appeared at the moment of impact. Des Coudres conducted experiments in the 1890's [17, 18] in which an electric potential difference was detected between the center and outer edge of a rotating cylinder of electrolyte.

The studies of Lodge, Colley and Des Coudres concerned the masses of the ionic charge carriers in electrolytes. (Inconclusive) efforts were made by Nichols (1906) [19] to observe an electric potential difference in a rotating metal cylinder, where electrons are the charge carriers. This effect has still not been conclusively observed for electrons.

Tolman (1913-1926) [22]-[26] followed the suggestion of Maxwell in Art. 574 of [10] that if a current-carrying coil is mounted as a torsion pendulum, and the current is suddenly

[4] An extensive discussion of the historical context of Thomson's measurement is given in [6].
[5] From Art. 377 of [7], now known as Faraday's first law of electrolysis.
[6] For historical reviews, see [8, 9].

stopped, a small, transient deflection of the pendulum should be observable due to the torque associated with disappearance of the mechanical angular momentum of the current. This effect was observed, although not very accurately.[7]

The converse effect, also considered by Maxwell in Art. 574 of [10], that a (small) torque would be required to generate the mechanical angular momentum of a loop of electric current as the current increases, was finally observed by Barnett (1931) [28]. This was part of a larger effort [29] to measure the gyromagnetic ratio of the electron, $\Gamma = \mu/L$, where μ is the magnetic moment and L is the angular momentum. A classical view of an electron of charge e and mass m moving in a circle of radius r with velocity v is that $\mu = IA = (ev/2\pi r)(\pi r^2) = (e/2m)(mvr) = (e/2m)L$, such that $\Gamma = e/2m$, as perhaps first clearly noted in [30, 31]. The first reported measurement of Γ, by Einstein and de Haas (1915) [30], claimed the result to be the classical value, while Barnett (1915) [31] reported twice that, as eventually confirmed by him and others.

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