

Wedgfall

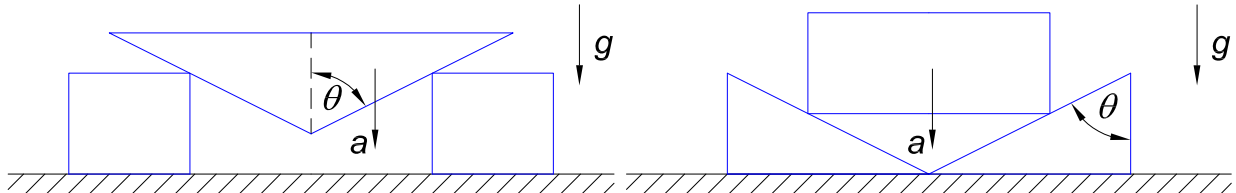
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1 Problem

Deduce the vertical acceleration a of a wedge of vertical angle θ and mass $2m$ that is in contact with two rectangular blocks of mass m each, as shown in the left sketch below, assuming no friction anywhere. As the wedge falls vertically, the block is accelerated horizontally.



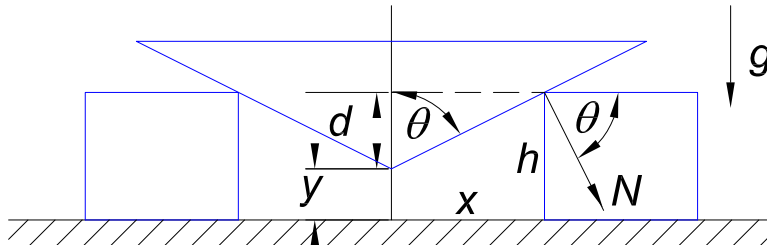
What is the vertical acceleration a of a block of mass $2m$ that is in contact with two wedges of mass m and vertical angle θ , as shown in the right sketch below, again assuming no friction anywhere?

2 Solution

This problem is related to the well-known example of a rectangular block that slides without friction on a wedge that slides on a horizontal plane. A more complex variation has been posed at Phys. Teach. 56, 527 (2013).

2.1 Falling Wedge

The key to this problem is the establishment of the kinematic constraint between the vertical motion of the wedge with acceleration $a = a_y$ (positive downwards) and the horizontal motion of the blocks with acceleration $\pm a_x$ (noting that horizontal momentum is conserved in this problem).



For this, we relate the vertical position y of the bottom tip of the wedge to the horizontal position x of the left edge of the right block relative to the symmetry axis, as shown in the figure below,

$$y + d = h, \quad \frac{x}{d} = \tan \theta, \quad y + \frac{x}{\tan \theta} = h, \quad a_y = \frac{a_x}{\tan \theta}, \quad (1)$$

since a_x is the acceleration of x while a_y is minus the acceleration of y .

To apply Newton's 2nd law to this problem, we note that the force of contact \mathbf{N} between the wedge and the block is normal to the surface of the wedge, and hence makes angle θ to the horizontal as shown above. The horizontal equation of motion of the right block is then

$$N \cos \theta = ma_x = ma_y \tan \theta, \quad (2)$$

while the vertical equation of motion of the wedge is

$$2mg - 2N \sin \theta = 2ma_y. \quad (3)$$

Thus,

$$mg = N \sin \theta + ma_y = m \tan^2 \theta a_y + ma_y, \quad (4)$$

$$a_y = \frac{g}{1 + \tan^2 \theta} = g \cos^2 \theta. \quad (5)$$

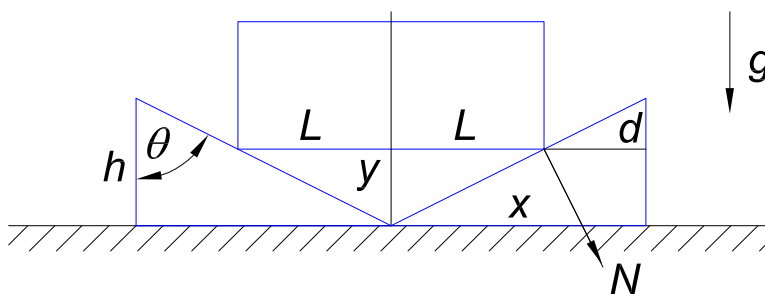
The limiting cases are $\theta = 0$ in which case the wedge is a narrow block that falls between the wall and the other block with $a_y = g$, and $\theta = 90^\circ$ in which case the wedge is a block that rests on the other block without moving ($a_y = 0$).

2.2 Falling Block

In this case, we relate the vertical position y of the bottom of the block (of total length $2L$) to the horizontal position x of the right edge of the right wedge relative to the symmetry axis, as shown in the figure above,

$$y = h - d, \quad \frac{x - L}{d} = \tan \theta, \quad y = h - \frac{x - L}{\tan \theta}, \quad a_y = \frac{a_x}{\tan \theta}, \quad (6)$$

since a_x is the acceleration of x while a_y is minus the acceleration of y .



To apply Newton's 2nd law to this problem, we note that the force of contact \mathbf{N} between the wedge and the block is normal to the surface of the wedge, and hence makes angle θ to the horizontal as shown above. The horizontal equation of motion of the right wedge is then

$$N \cos \theta = ma_x = ma_y \tan \theta, \quad (7)$$

while the vertical equation of motion of the block is

$$2mg - 2N \sin \theta = 2ma_y. \quad (8)$$

Thus,

$$mg = N \sin \theta + ma_y = m \tan^2 \theta a_y + ma_y, \quad (9)$$

$$a_y = \frac{g}{1 + \tan^2 \theta} = g \cos^2 \theta, \quad (10)$$

as for the case of a wedge falling between two blocks.