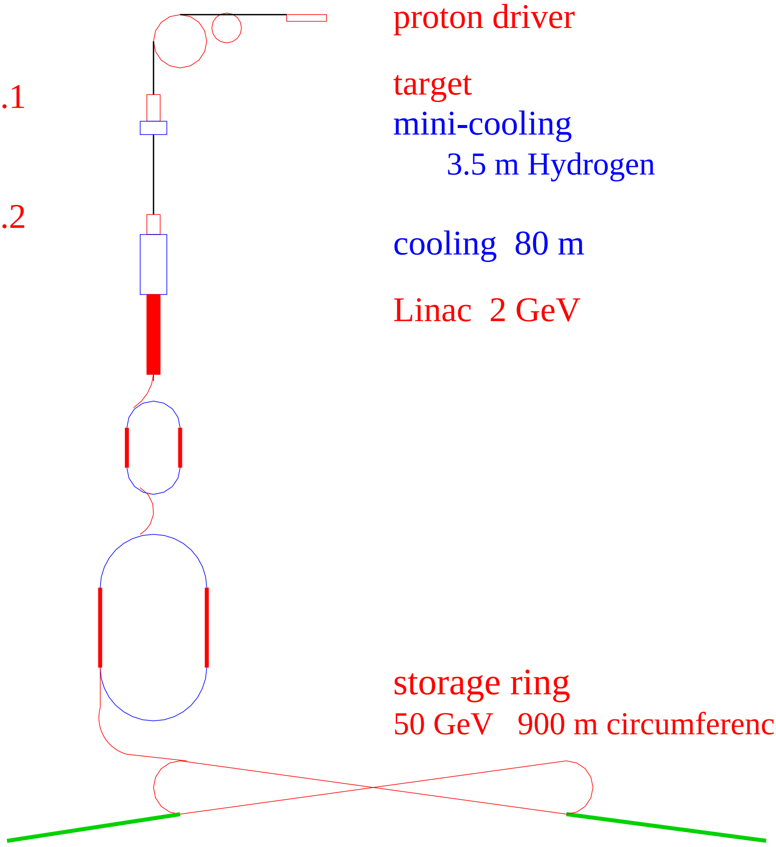
Physics Opportunities with Muon Beams:
Neutrino Factories and Muon Colliders



proton driver

target

mini-cooling
3.5 m Hydrogen

phase rotation No.1
42 m rf

drift 160 m

phase rotation No.2

cooling 80 m

Linac 2 GeV

recirculator Linac
2 - 8 GeV

recirculator Linac
8 - 50 GeV

storage ring
50 GeV 900 m circumference

neutrino beam

neutrino beam

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`http://puhep1.princeton.edu/~mcdonald/mumu/NSFLetter/`

Past Uses of Muon Beams:

- Measurement of $g-2$ of the muon.
- Search for "forbidden" processes: $\mu \rightarrow e\gamma$, $\mu N \rightarrow eN$, ...
- Study of nuclear structure via $\mu N \rightarrow \mu X$.

New Opportunities:

- Neutrino factories based on $\mu \rightarrow e\nu_\mu\bar{\nu}_e$.
  - **Neutrino oscillations.**
  - Nucleon structure via $\nu_\mu N \rightarrow \mu X$; X includes charm...
  - A path to muon colliders.
- Muon colliders.
  - s-channel production of light Higgs.
  - Precision studies of electroweak/supersymmetry physics.

    [Leptonic initial state;

    Beamstrahlung suppressed by $(m_e/m_\mu)^2$.]
  - A new path to the energy frontier.

Oscillations of Massive Neutrinos

Neutrinos could have a small mass (Pauli, Fermi, Majorana, 1930's).

Massive neutrinos can mix (Pontecorvo, 1957).

In the example of only two massive neutrinos, with mass eigenstates ν_1 and ν_2 with mass difference Δm and mixing angle θ, the flavor eigenstates ν_a and ν_b are related by

$$\begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

The probability that a neutrino of flavor ν_a and energy E appears as flavor ν_b after traversing distance L in vacuum is

$$P(\nu_a \to \nu_b) = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2[\mathrm{eV}^2]\ L[\mathrm{km}]}{E[\mathrm{GeV}]} \right).$$

The probability that ν_a does not disappear is

$$P(\nu_a \to \nu_a) = \cos^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2[\mathrm{eV}^2]\ L[\mathrm{km}]}{E[\mathrm{GeV}]} \right).$$

A Sketch of Current Data

- The "anomaly" of atmospheric neutrinos suggests that GeV ν_μ's disappear while traversing the Earth's diameter.
  $\Rightarrow \Delta m^2 \approx 10^{-3}\ (\mathrm{eV})^2$ for $\sin^2 2\theta \approx 1$.
  (Kamiokande, IMB, Soudan-2, MACRO, Super-Kamiokande)

- The solar neutrino "deficit" suggests that MeV ν_e's disappear between the center of the Sun and the Earth.
  $\Rightarrow \Delta m^2 \approx 10^{-10}\ (\mathrm{eV})^2$ for $\sin^2 2\theta \approx 1$, if vacuum oscillations.
  (Homestake, GALLEX, SAGE)

- The LSND experiment at Los Alamos suggests that 30-MeV $\overline{\nu}_\mu$'s appears as $\overline{\nu}_e$'s after 30 m.
  $\Rightarrow \Delta m^2 \approx 1\ (\mathrm{eV})^2$, but reactor data requires $\sin^2 2\theta \lesssim 0.03$.

The first two results require at least 3 massive neutrinos.

All results together require at least 4 massive neutrinos.

The measured width of the Z^0 boson (LEP) $\Rightarrow$ only 3 Standard Model neutrinos. A 4th massive neutrino must be "sterile".

The Supersymmetric Seesaw

A provocative conjecture is that neutrino mass m_ν is coupled to two other mass scales, m_I (intermediate) and m_H (heavy), according to

$$m_\nu = \frac{M_I^2}{M_H}.$$

(Gell-Mann, Ramond, Slansky, 1979)

A particularly suggestive variant takes $m_I = \langle \phi_{\mathrm{Higgs}} \rangle = 250$ GeV; Then

$m_\nu \approx \sqrt{\Delta m^2(\mathrm{atmospheric})} \approx 0.06\ \mathrm{eV} \Rightarrow m_H \approx 5 \times 10^{15}\ \mathrm{GeV}.$

This is perhaps the best experimental evidence for a grand unification scale, such as that underlying supersymmetric SO(10) models.

Neutrino oscillations $\overset{?}{\Rightarrow}$ Supersymmetry.

Mixing of Three Neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13}e^{-i\delta} \\ -\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{13}\mathrm{s}_{23}e^{i\delta} & \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{13}\mathrm{s}_{23}e^{i\delta} & \mathrm{c}_{13}\mathrm{s}_{23} \\ \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{s}_{13}\mathrm{c}_{23}e^{i\delta} & -\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{s}_{13}\mathrm{c}_{23}e^{i\delta} & \mathrm{c}_{13}\mathrm{c}_{23} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where $\mathrm{c}_{12} = \cos\theta_{12}$, *etc.* (Maki, Nakagawa, Sakata, 1962).

Three massive neutrinos ⇒ six independent parameters:

- Three mixing angles: θ_{12}, θ_{13}, θ_{23},
- A phase δ related to CP violation,
- Two differences of the squares of the neutrino masses.
  Ex: $\Delta m^2_{12} = \Delta m^2(\text{solar})$ and $\Delta m^2_{23} = \Delta m^2(\text{atmospheric})$.

Measurement of these parameters is a primary goal of experimental neutrino physics.

If four massive neutrinos, then 6 mixing angles, 3 phases,
3 independent squares of mass differences.

[Theorists find the MNS matrix more analyzable than the CKM matrix.]

Matter Effects

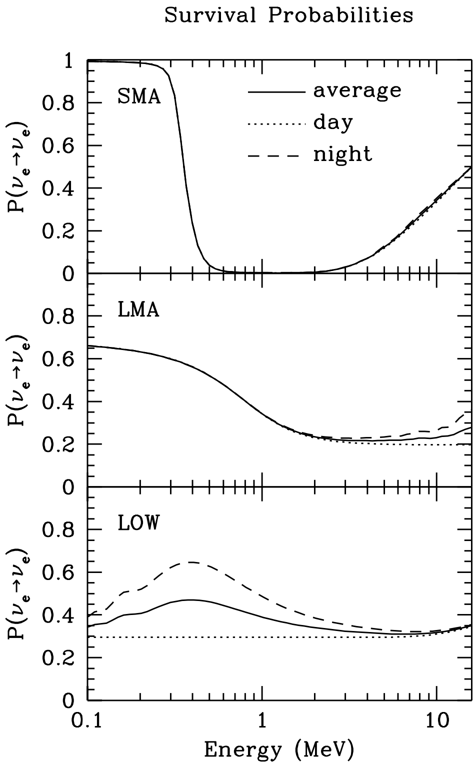
ν_e's can interact with electrons via both W and Z^0 exchanges, but other neutrinos can only interact via Z^0 exchange.

$$\Rightarrow \sin^2 2\theta_{\text{matter}} = \frac{\sin^2 2\theta_{\text{vac}}}{\sin^2 2\theta_{\text{vac}} + (\cos 2\theta_{\text{vac}} - A)^2},$$

where $A = 2\sqrt{2} G_F N_e E/\Delta m^2$ depends on sign of Δm^2.

At the "resonance", $\cos 2\theta_{\text{vac}} = A$, $\sin^2 2\theta_{\text{matter}} = 1$ even if $\sin^2 2\theta_{\text{vac}}$ is small (Wolfenstein, 1978, Mikheyev, Smirnov, 1986).

$\Rightarrow$ 3 MSW solutions to the solar neutrino problem:



Too Many Solutions

There are 8 scenarios suggested by present data:

- Either 3 or 4 massive neutrinos.
- Four solutions to the solar neutrino problem:
  1. Vacuum oscillation (VO) solution;
     $\Delta m^2_{12} \approx (0.5 - 5.0) \times 10^{-10}$ eV^2, $\sin^2 2\theta_{12} \approx (0.7 - 1.0)$.
  2. Low (Just So) MSW solution;
     $\Delta m^2_{12} \approx (0.5 - 2.0) \times 10^{-7}$ eV^2, $\sin^2 2\theta_{12} \approx (0.9 - 1.0)$.
  3. Small mixing angle (SMA) MSW solution;
     $\Delta m^2_{12} \approx (4.0 - 9.0) \times 10^{-6}$ eV^2, $\sin^2 2\theta_{12} \approx (0.001 - 0.01)$.
  4. Large mixing angle (LMA) MSW solution;
     $\Delta m^2_{12} \approx (0.2 - 2.0) \times 10^{-4}$ eV^2, $\sin^2 \theta_{12} \approx (0.65 - 0.96)$.
- Atmospheric neutrino data $\Rightarrow \Delta m^2_{23} \approx (3 - 5) \times 10^{-4}$ eV^2, $\sin^2 \theta_{12} > 0.8$.
- θ_{13} very poorly known; δ completely unknown.

The Next Generation of Neutrino Experiments

- Short baseline accelerator experiments (miniBoone, ORLAND, CERN) will likely clarify the LSND result.

- Super-Kamiokande + new long baseline accelerator experiments (K2K, Minos, CERN) will firm up measurements of θ_{23} and Δm^2_{23}, but will provide little information on θ_{13} and δ.

- New solar neutrino experiments (BOREXino, SNO, HELLAZ, HERON, ...) will explore different portions of the energy spectrum, and clarify possible pathlength-dependent effects.

  SNO should provide independent confirmation of neutrino oscillations via comparison of reactions $\nu + {}^2\mathrm{H} \rightarrow p + p + e$ and $\nu + {}^2\mathrm{H} \rightarrow p + n + \nu$.

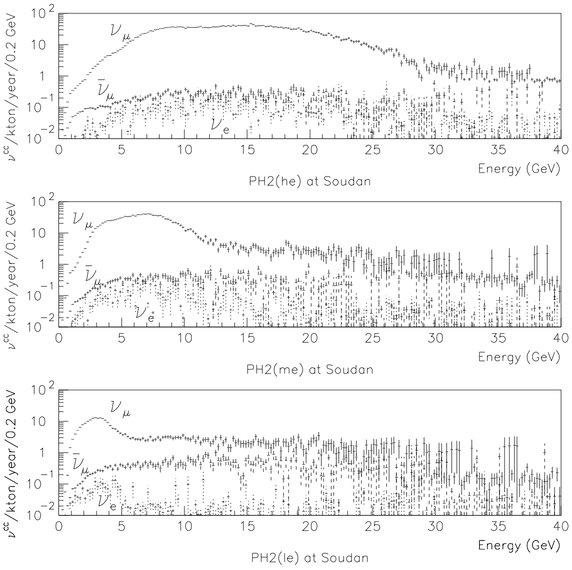
- Each of these experiments studies oscillations of only a single pair of neutrinos.

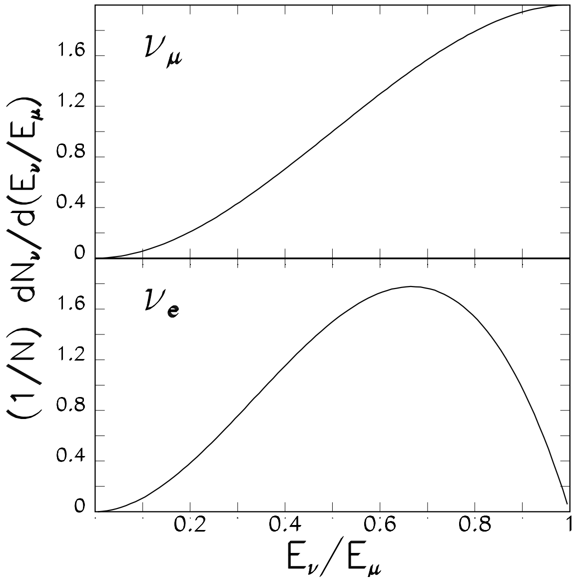
- The continued search for the neutrinoless double-beta decay ${}^{78}\mathrm{Ge} \rightarrow {}^{78}\mathrm{Se} + 2e^-$ will improve the mass limits on Majorana neutrinos to perhaps as low as 0.001 eV (hep-ex/9907040).

The Opportunity for a Neutrino Factory

- Many of the neutrino oscillation solutions permit study of the couplings between 2, 3, and 4 neutrinos in accelerator based experiments.
- More neutrinos are needed!
- Present neutrino beams come from $\pi, K \rightarrow \mu\nu_\mu$ with small admixtures of $\overline{\nu}_\mu$ and ν_e from μ and $K \rightarrow 3\pi$ decays.
- Higher (per proton beam power), and better characterized, neutrino fluxes are obtained from μ decay.

  Collect low-energy μ's from π decay, accelerate the μ's to the desired energy, and store in a ring while they decay via $\mu^- \rightarrow e^-\nu_\mu\overline{\nu}_e$. [Of course, can use μ^+ also.]





6 Classes of Experiments at a Neutrino Factory

$$\nu_\mu \to \nu_e \to e^- \qquad \text{(appearance)}, \tag{1}$$

$$\nu_\mu \to \nu_\mu \to \mu^- \qquad \text{(disappearance)}, \tag{2}$$

$$\nu_\mu \to \nu_\tau \to \tau^- \qquad \text{(appearance)}, \tag{3}$$

$$\overline{\nu}_e \to \overline{\nu}_e \to e^+ \qquad \text{(disappearance)}, \tag{4}$$

$$\overline{\nu}_e \to \overline{\nu}_\mu \to \mu^+ \qquad \text{(appearance)}, \tag{5}$$

$$\overline{\nu}_e \to \overline{\nu}_\tau \to \tau^+ \qquad \text{(appearance)}. \tag{6}$$

[Plus 6 corresponding processes for $\overline{\nu}_\mu$ from μ^+ decay.]

Processes (2) and (5) are easiest to detect, via the final state μ.

Process (5) is noteworthy for having a "wrong-sign" μ.

Processes (3) and (6) with a final state τ require μ's of 10's of GeV.

Processes (1) and (4) with a final state electron are difficult to detect.

Finely segmented, magnetic detectors of 10's of kilotons will be required.

The Rates are High at a Neutrino Factory

Charged current event rates per kt-yr.

($L = 732$ km)	ν_μ	$\overline{\nu}_e$
Neutrino Factory	(2×10^{20} ν_μ/yr)	
10 GeV	2200	1300
20 GeV	18,000	11,000
50 GeV	2.9×10^5	1.8×10^5
250 GeV	3.6×10^7	2.3×10^7
MINOS (WBB)		
Low energy	460	1.3
Medium energy	1440	0.9
High energy	3200	0.9

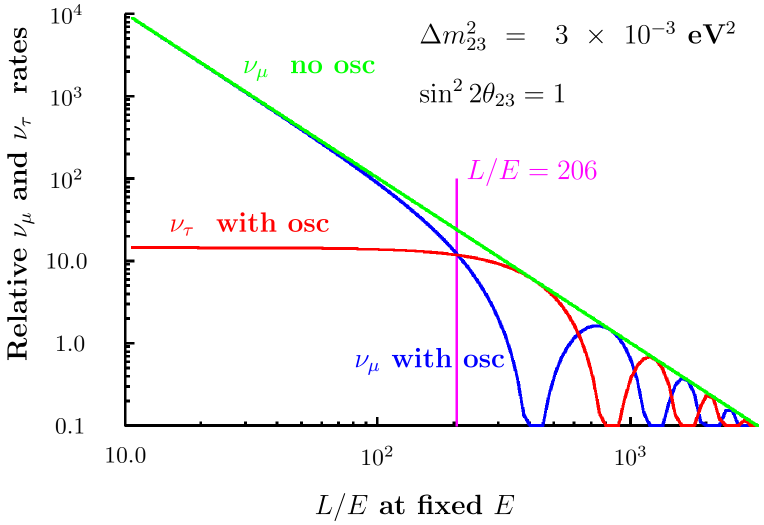
Even a low-energy neutrino factory has high rates of electron neutrino interactions.

A neutrino factory with $E_\mu \gtrsim 20$ GeV is competitive for muon neutrino interactions.

Scaling Laws for Rates at a Neutrino Factory

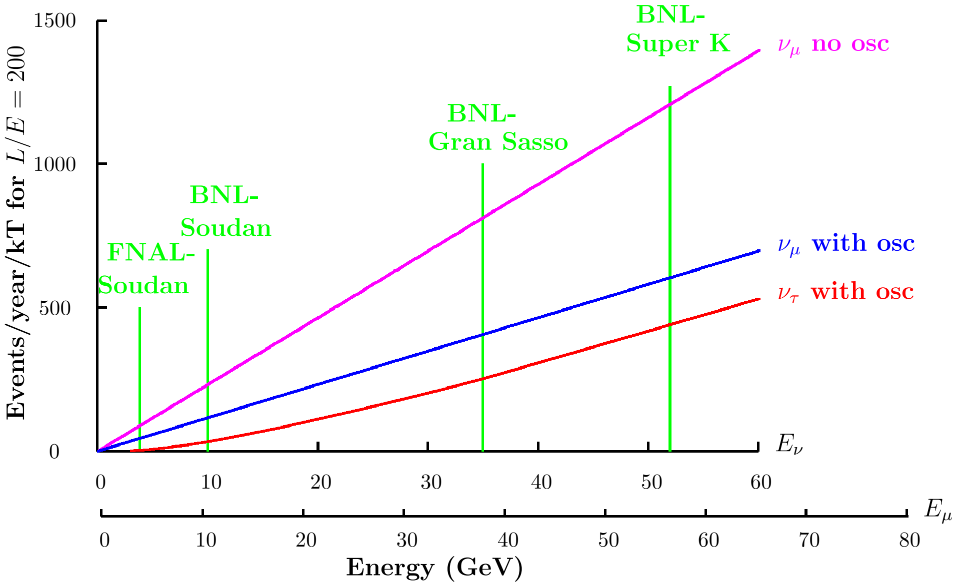
$\sigma_\nu \propto E$; $I_\nu \propto 1/(\theta)^2 \propto (E/L)^2$: Rate $\propto I_\nu \sigma_\nu \propto E^3/L^2$.

$\Rightarrow$ Rate $\propto E^3$ at fixed L, Rate $\propto 1/L^2$ at fixed E.



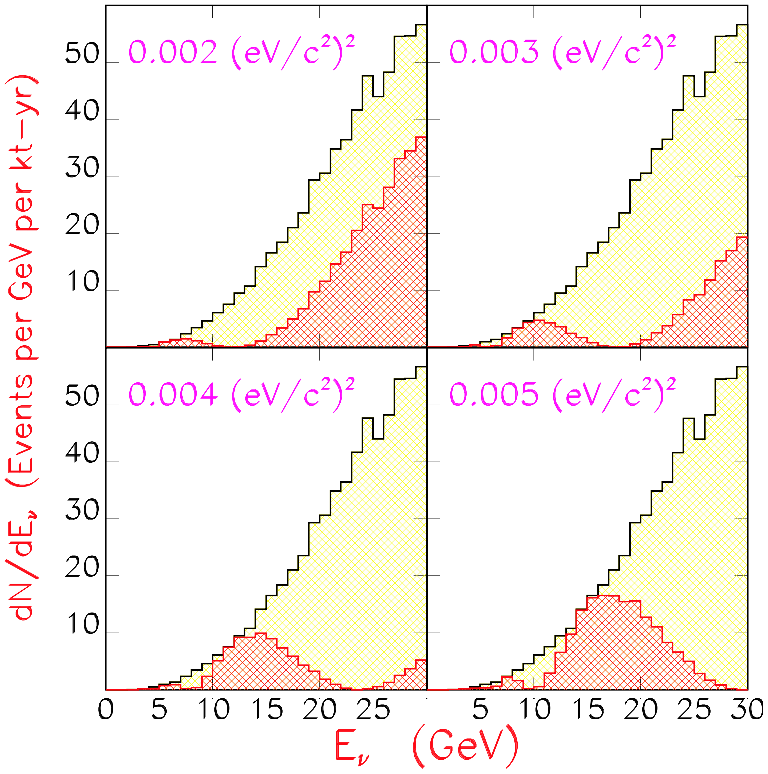
Neutrino oscillation probability varies with L/E,

$\Rightarrow$ Rate $\propto E$ for fixed L/E.



τ appearance suppressed at low energy. Larger $E \Rightarrow$ larger L.

$\nu_\mu \rightarrow \ \nu_\mu \rightarrow \mu^-$ **Disappearance**



$E_\mu = 30$ GeV,
2×10^{20} μ decays,
$L = 7000$ km,
$\sin^2 2\theta_{23} = 1$.
(hep-ph/9906487)

Δm^2_{23} (eV^2)	Events (per 10 kt-yr)
0.002	2800
0.003	1200
0.004	900
0.005	1700
No Osc.	6200

$\nu_\mu \rightarrow \ \nu_\tau \rightarrow \tau^-$ **Appearance**

Δm^2_{23} (eV^2)	Events (per 10 kt-yr)
0.002	1200
0.003	1900
0.004	2000
0.005	1800

For conditions as above.

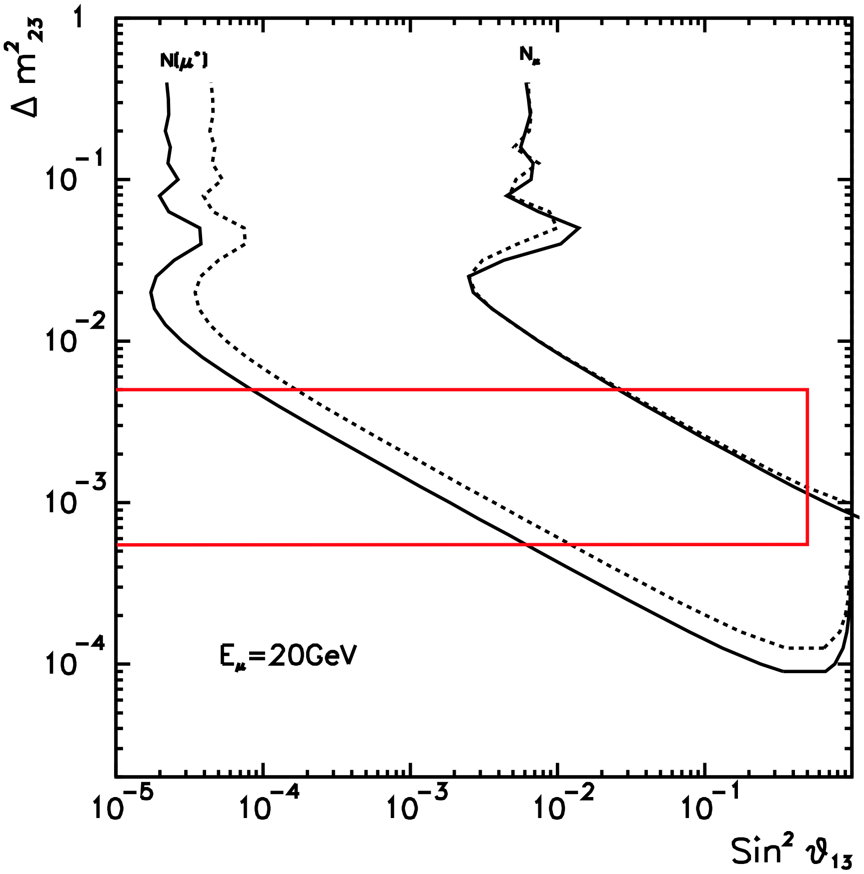
Measuring θ_{13}

Many ways:

$$P(\overline{\nu}_e \to \overline{\nu}_\mu) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \frac{1.27\Delta m^2_{23} L}{E_\nu},$$

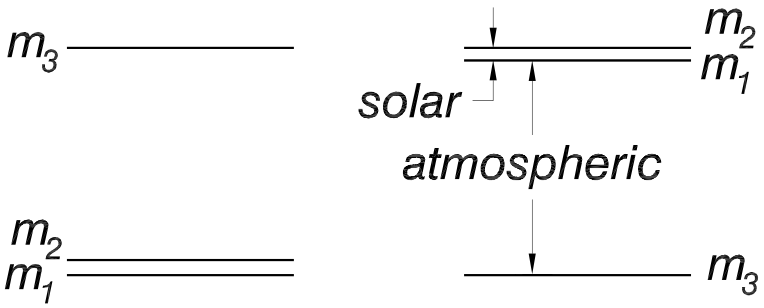
$$P(\overline{\nu}_e \to \overline{\nu}_\tau) = \sin^2 2\theta_{13} \cos^2 \theta_{23} \sin^2 \frac{1.27\Delta m^2_{23} L}{E_\nu},$$

$$P(\nu_\mu \to \nu_\tau) = \cos^4 \theta_{13} \sin^2 2\theta_{23} \sin^2 \frac{1.27\Delta m^2_{23} L}{E_\nu}.$$



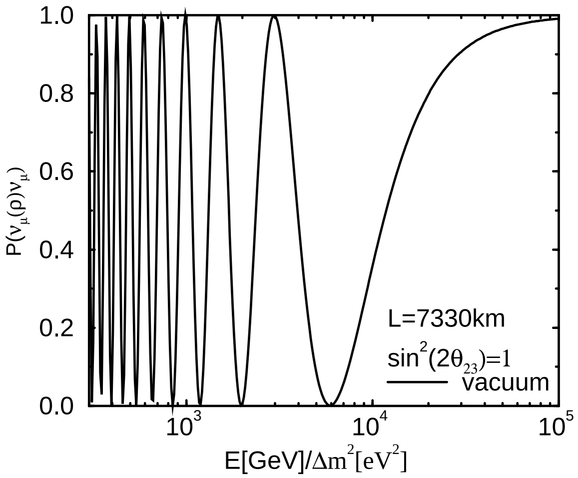
10 kton detector,
$E_\mu = 20$ GeV,
2×10^{20} μ decays,
$L = 732$ km,
$\sin^2 2\theta_{23} = 1$,
Left: $\overline{\nu}_e \to \overline{\nu}_\mu \to \mu^+$,
Right: $\nu_\mu \to \nu_\mu \to \mu^-$,
Box = presently allowed.
(hep-ph/9811390).

Measuring the Sign of Δm^2_{23} via Matter Effects

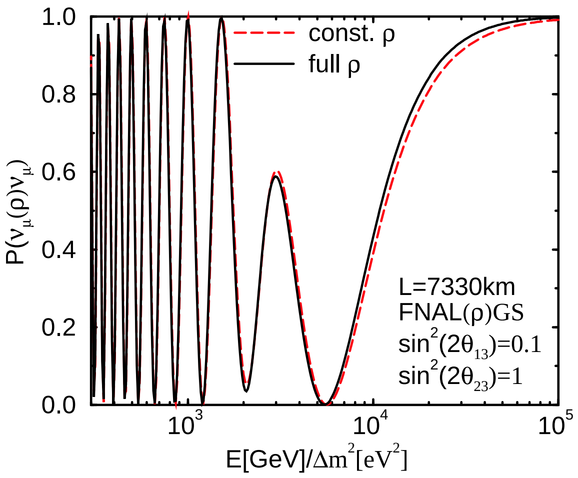


The matter effect resonance depends on the sign of Δm^2 (p. 7).

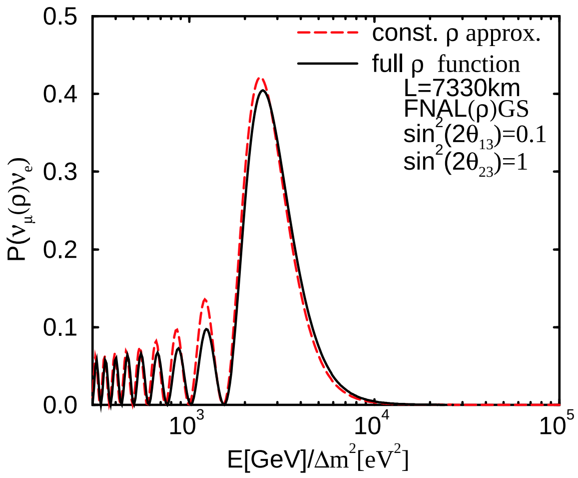
Large effect of Δm^2_{23} in ν_μ (disappearance) if $\sin^2 2\theta_{13} \approx 0.1$.

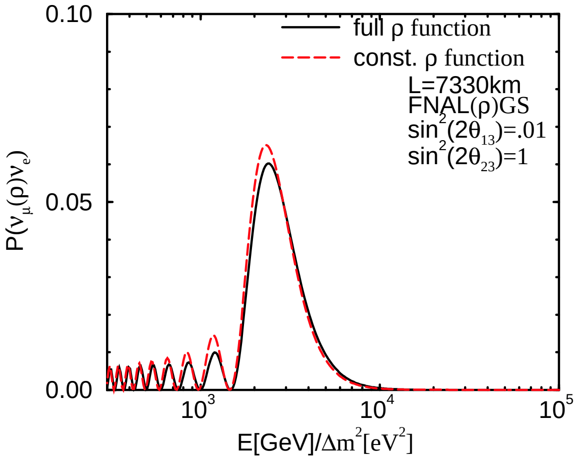


(hep-ph/ 9910554)



For smaller $\sin^2 2\theta_{13}$, may be better to use $\overline{\nu}_e \rightarrow \overline{\nu}_\mu$ (appearance).





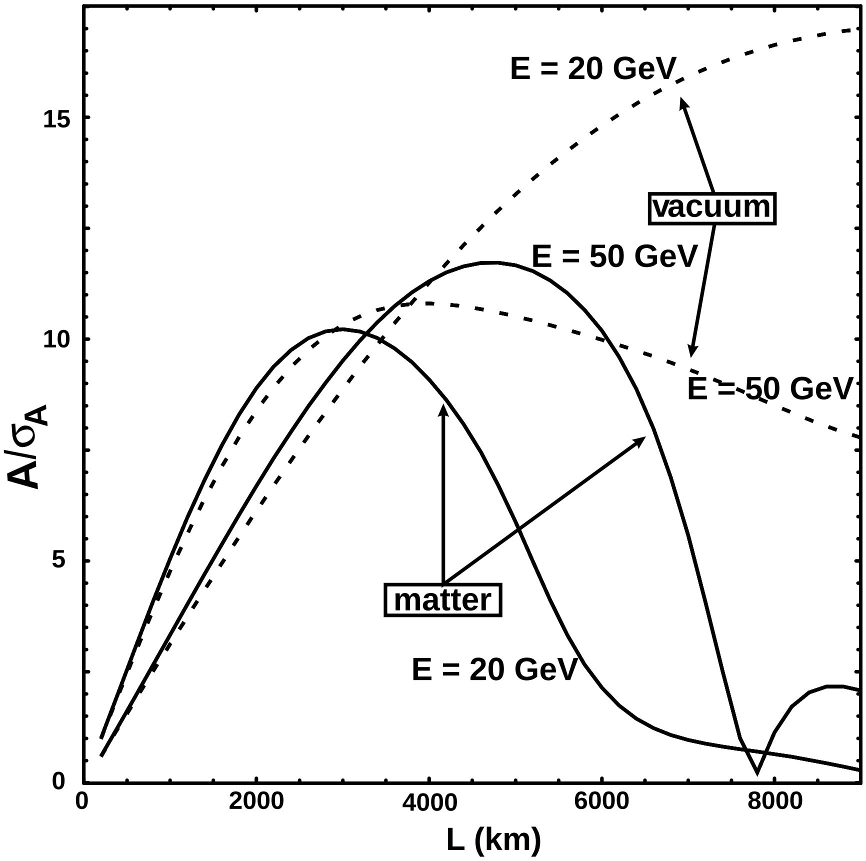
Measuring δ via CP Violation

The phase δ is accessible to terrestrial experiment in the large mixing angle (LMA) solution to the solar neutrino problem (or if there are 4 massive neutrinos).

CP violation:

$$A_{\mathrm{CP}} = \frac{P(\nu_e \to \nu_\mu) - P(\overline{\nu}_e \to \overline{\nu}_\mu)}{P(\nu_e \to \nu_\mu) + P(\overline{\nu}_e \to \overline{\nu}_\mu)} \approx \left| \frac{2\sin\delta}{\sin 2\theta_{13}} \sin \frac{1.27\Delta m^2_{12} L}{E} \right|,$$

assuming $\sin^2 2\theta_{12} \approx \sin^2 2\theta_{23} \approx 1$ (LMA).



10 kton detector,
2×10^{21} muon decays,
Large angle MSW:
$\Delta m^2_{12} = 10^{-4}$ eV^2,
$\Delta m^2_{23} = 2.8 \times 10^{-3}$ eV^2,
$\theta_{12} = 22.5°$,
$\theta_{13} = 13°$,
$\theta_{23} = 45°$,
$\delta = -90°$.
(hep-ph/9909254)

Matter effects dominate the asymmetry for $L > 1000$ km.

Measuring δ via T Violation

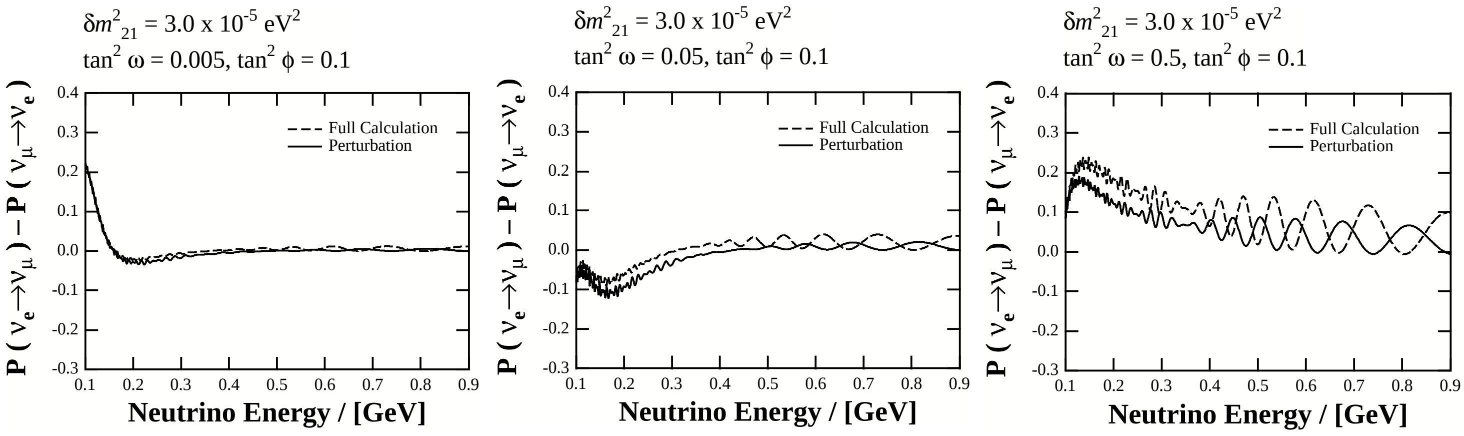
If the small mixing angle (SMA) solutions holds, may still be able to measure δ via T violation:

$$P(\nu_e \rightarrow \ \nu_\mu) - P(\nu_\mu \rightarrow \ \nu_e) =$$

$$4J\left(\sin\frac{1.27\Delta m^2_{12}L}{E} + \sin\frac{1.27\Delta m^2_{13}L}{E} + \sin\frac{1.27\Delta m^2_{23}L}{E}\right),$$

$$J = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\mathrm{sin}\,\delta = \text{Jarlskog invariant.}$$

Matter effects could make $\sin 2\theta_{12}$ resonance for $E \approx 100$ MeV and $L \approx 10,000$ km (hep-ph/9911258).



However, not easy to measure $\nu_\mu \rightarrow \nu_e \rightarrow e^-$ (appearance) against background of $\overline{\nu}_e \rightarrow \overline{\nu}_e \rightarrow e^+$ in a large, massive detector in which the electrons shower immediately. [Rates low also.]

Controlling the ν_e Flux via Muon Polarization

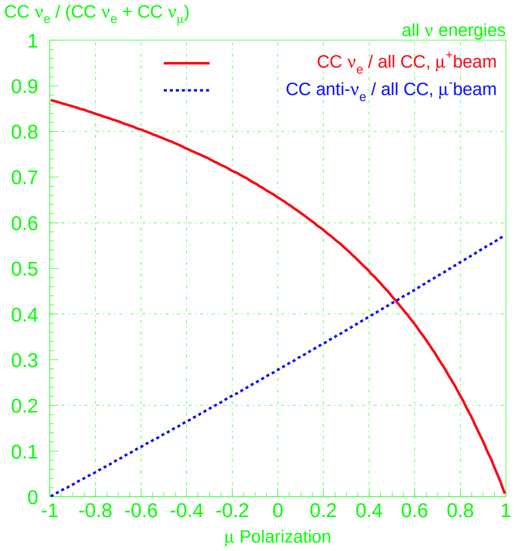
For μ^- decay in flight,

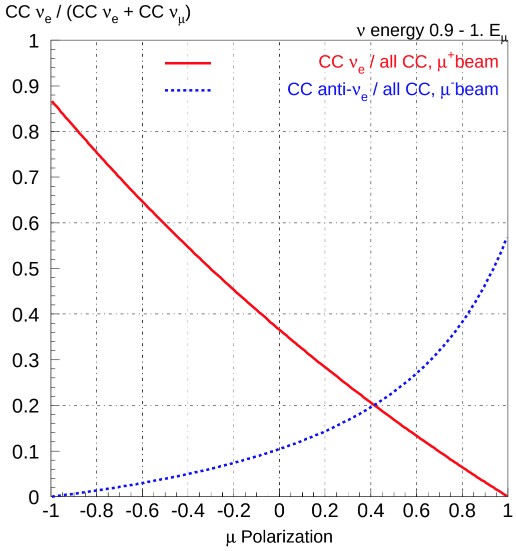
$$\frac{dN_{\nu_\mu}(\theta_{\nu_\mu}=0)}{dx} = 2Nx^2[(3-2x)+P(1-2x)],$$
$$\frac{dN_{\overline{\nu}_e}(\theta_{\overline{\nu}_e}=0)}{dx} = 12Nx^2(1-x)(1+P),$$

where $x = 2E_\nu/m_\mu$, and P is the muon polarization.

[$\theta_\nu = 0 \Rightarrow$ colinear decay; at $P=-1$, all colinear decays forbidden for $\theta_{\nu_e}=0$, but one is allowed for $\theta_{\nu_\mu}=0$.]

Modulate the muon polarization to modulate the relative rates of $\nu_\mu \to \nu_e \to e^-$ and $\overline{\nu}_e \to \overline{\nu}_e \to e^+$.





(Blondel, http://alephwww.cern.ch/~bdl/muon/nufacpol.ps)

Summary

- The physics program of a neutrino factory/muon collider is extremely diverse, and of scope to justify an international laboratory.
- The first step is a neutrino factory capable of systematic exploration of neutrino oscillations.
  - With $\gtrsim 10^{20}$ ν's/year can go well beyond other existing or planned accelerator experiments.
  - Beams with $E_{\nu_e} \lesssim 1$ GeV are already very interesting.
  - Higher energy is favored: Rate $\propto E$ at fixed L/E; ν_τ appearance practical only for $E \gtrsim 30$ GeV.
  - Detectors at multiple distances needed for broad coverage of parameter space $\Rightarrow$ triangle or "bowtie" storage rings.
  - CP and T violation accessible with $\gtrsim 10^{21}$ ν's/year.
  - Control of muon polarization extremely useful when studying $\nu_e \to e$ modes.