The Cyclic Theory of the Universe

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Abstract

The cyclic theory of the universe is a radical alternative to the standard big bang/inflationary scenario that offers a new approach for resolving the homogeneity, isotropy, and flatness problems and generating a nearly scale-invariant spectrum of fluctuations. The original formulation of the cyclic model was based on the picture suggested by M-theory in which the observable universe lies on a brane separated by a small gap along an extra dimension from a second brane. The cyclic model proposes that the big bang is a collision between branes that occurs at regular intervals; that each bang creates hot matter and radiation and triggers an epoch of expansion, cooling and structure formation; that there is an interbrane force responsible for drawing the branes together whose potential energy acts like dark energy when the branes are far apart; and that each cycle ends with the contraction of the extra dimension and a collision between branes – a new big bang – that initiates the next cycle. In more recent formulations, the cyclic model is realized with ordinary quantum field theory without introducing branes or extra dimensions. The key innovation common to all these models is the *ekpyrotic phase*, the period of ultra-slow contraction preceding the big bang. It is the ekpyrotic phase, rather than inflation, that is responsible for explaining the smoothness, flatness and large scale structure of the universe. It is also the ekpyrotic phase that generates the distinctive signatures in the spectrum of primordial gravitational waves and non-gaussian density fluctuations that will be used to test the cyclic model in forthcoming experiments.

1 Motivation

The cyclic theory of the universe [Steinhardt and Turok(2002a), Steinhardt and Turok(2002b)] challenges the standard big bang/inflationary picture [Guth(1981), Linde(1982), Albrecht and Steinhardt(1982)] by offering an alternative explanation for the homogeneity, isotropy and flatness of the universe, the absence of magnetic monopoles and other massive stable relics from the Planckian era, as well as the generation of a nearly scale-invariant spectrum of density fluctuations. The theory is based on three underlying notions: (1) the big bang is not the beginning of space and time, but rather a transition from an earlier phase of evolution; (2) big bangs occurred periodically in the past and continue periodically into the future; and, (3) the key events that shaped the large scale structure of the universe occurred during a phase of slow contraction before the big bang. The third item contrasts with the standard big bang picture in which the large scale structure of the universe is shaped by a period of ultra-rapid expansion (inflation) that occurs strictly after the bang. The cyclic theory eschews inflation altogether in favor of an *ekpyrotic* phase of ultra-slow contraction before the bang during which $w \gg 1$ (where w is the ratio of pressure to energy density). The big surprise and a compelling reason to take the cyclic theory seriously is the discovery that an inflationary expanding phase with $w \approx -1$ produces almost exactly the same large scale structure as an ekpyrotic contraction phase with $w \gg 1$, to the degree that it is impossible to distinguish the two pictures based on current observations. Yet, the two pictures provide extraordinarily different visions of space, time, and the global structure of the universe, and are ultimately distinguishable through future observations.

There are several important reasons for seeking an alternative to inflation. First, exploring alternatives is an effective way of determining if current observations are sufficient to specify uniquely the cosmic history of the universe. The example of the cyclic model teaches us that that answer is no. It also points us to the open issues theorists and experimentalists need to explore to move the field forward.

Second, although the inflationary model was developed over two decades ago, its weaknesses have never been successfully addressed. Twenty years ago, the pressing questions were: What is the inflaton and why are its interactions finely-tuned? How did the universe begin and why did it lead to inflation? Those same questions remain today. The hope had been that an improved approach to fundamental physics, such as string theory, might answer these questions. Yet, despite heroic efforts to construct stringy inflation models with tens or hundreds of moving parts (fluxes, throats and branes) and to consider a complex landscape of 10^{500} vacua, no compelling inflationary model has been found [Maldacena(2003)]. The most carefully studied examples to date (d-brane inflation in warped conifolds) suggest that inflation is typically spoiled by quantum corrections and topological constraints and that the few surviving examples require delicate tuning of both parameters and initial conditions [Baumann et al.(2007a)Baumann, Dymarsky, Klebanov, McAllister, and Steinhardt]. Other approaches to string inflation have not been critically examined to this degree, but there is no reason to expect that they will be immune.

The discovery of dark energy is another setback for inflation. The hope had been that, by adding inflation to the big bang picture, the rest of the history of the universe would be set. We now know that, at best, inflation fixes the next 9 billion years. The rest of the history of the universe is dominated by dark energy, whose origin, stability and other physical properties are unknown. The standard big bang/inflationary picture appears to be put together piecemeal from separately adjustable components the big bang, inflation and, more recently, dark energy that are not directly linked to one another. This reduces the overall coherence and explanatory power of the theory.

Perhaps the strongest motivation for considering alternatives to inflation is that the inflationary model may not have the powerful predictive power it was originally thought to have. The problem derives from the role of quantum physics plays in inflationary cosmology. On the one hand, quantum effects are essential for terminating inflation and for obtaining a nearly scale-invariant spectrum of nearly gaussian fluctuations, as required to match observations. On the other hand, along with these appealing features come undesirable consequences: First, because of its quantum instability, inflation is geodesically incomplete to the past; this means that the theory is incomplete and its predictions uncertain without some explanation for how the universe emerged from the big bang and why inflation was initiated. Second, rare quantum fluctuations prolong inflation eternally and dominate the volume of the universe; consequently, instead of approaching uniformity, the universe is actually remains inhomogeneous on superhorizon scales with only rare pockets of space looking like what astronomers and physicists observe. The alarming corollary is the *unpredictability problem*: If inflation is eternal, then, due to random quantum fluctuations, there are an infinite number of flat bubbles, but also an infinite number of open bubbles; there are an infinite number with nearly scale-invariant spectra, but also an infinite number with different spectra. Unless some rigorous method can be found for determining which kind of pocket is more likely – and there are arguments to suggest this cannot be done any outcome is possible and the famous inflationary predictions ought to be retracted.

All of this serves as motivation for considering the cyclic theory of the universe, a radical proposal for addressing these issues and more in an efficient, integrated picture. The next section provides a basic overview. Then, Section 3 focuses on the most novel aspect of the cyclic theory, the ekpyrotic contraction phase, which replaces inflation in setting the large scale structure of the universe. Sections 4 and 5 delve into the key predictions of the cyclic theory that arise from the ekpyrotic phase, the spectrum of scalar and tensor perturbations, respectively. Of special note are Sections 4.3 and 5 that summarize the predictions themselves, including the predictions of non-gaussianity and gravitational waves that distinguish the cyclic model from inflation. Section 6 addresses one of the most common questions about the cyclic model: namely, is the cyclic model compatible with the laws of thermodynamics and the prohibition against perpetual motion? In the final section, we turn to many of the criticisms of the cyclic model that have been most often cited and point out how the theoretical progress presented in the previous sections answers them. The chapter closes with a discussion of the key remaining open issue: precisely how the bounce from big crunch to big bang occurs.

2 Overview of the Cyclic Picture

The cyclic model is motivated by recent developments in string theory, especially the notions of orbifold planes (branes), extra dimensions, and their manifestation in Horava-Witten and heterotic M-theory [Horava and Witten(1996), Lukas et al.(1999a)Lukas, Ovrut, Stelle, and Waldram, Lukas et al.(1999b)Lukas, Ovrut, and Wa
According to this picture, our visible three (spatial) dimensions correspond to one of the branes, which is separated by a tiny gap across an extra, hidden dimension from a similar, though not identical brane. In M-theory, there are also six additional hidden dimensions associated with each point that are wrapped in a Calabi-Yau manifold. We For the most part, these six dimensions play no active role (though see Sec. 4) so we will ignore them and treat the cyclic picture as if it is 5d (four spatial dimensions plus time).

The brane picture provides a view of the cyclic theory that is geometric and very intuitive, as illustrated in Fig. (2). The cycles are due to the regularly repeating collision of two orbifold planes along the extra spatial dimension in which each collision corresponds to a kind of big bang. With each bang, new matter and radiation is created and a new period of expansion (stretching of the branes in the three large dimensions) begins. The collision occurs at finite speed and energy density, so the maximum temperature after the bang is finite, estimated to be about 10^{23} K (based on the condition that the density fluctuation amplitude on large scales should match observations [Khoury et al.(2004)Khoury, Steinhardt, and Turok]). The production of density fluctuations is due to quantum fluctuations that wrinkle the branes as they approach one another, causing the collision across the branes to occur at slightly different times, leading to spatial variations in the temperature and density. The branes bounce apart after the collision and begin to stretch, causing the hot gas of matter and radiation to expand and cool. Dark energy is the potential energy due to the interbrane force that attracts the branes to one another and causes them to collide on a regular basis, perhaps once every trillion years or so. The dark energy phase is naturally terminated as the interbrane force draws the branes together, when it is transformed into brane kinetic energy. Some of this kinetic energy is transformed into matter and radiation at the collision. The remaining energy combined with energy from the gravitational field suffice to bring the branes back to the same positions they occupied a cycle earlier, before the branes began to move. And then the cycle begins anew.

Although some may find the colliding brane picture compelling, others may be skeptical because it appears to rely on the reality of branes, extra dimensions, and string theory, which are unproven. It is

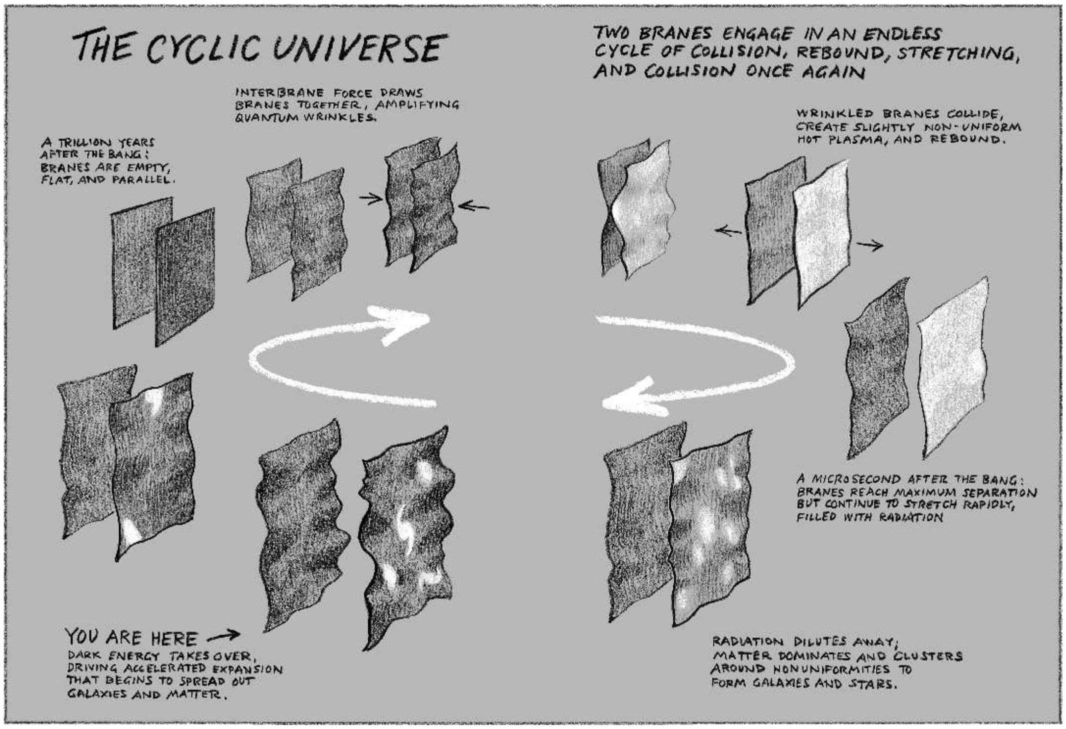

Figure 1: A schematic illustration of the colliding brane picture of the cyclic theory beginning from the present epoch (marked "You are here). [Figure reprinted by permission of authors from *Endless Universe*, by P.J. Steinhardt and N. Turok, (Doubleday, 2007).

important to appreciate, therefore, that most and perhaps all of the features of the colliding brane picture can be mimicked with ordinary quantum fields in three spatial dimensions (plus time). This "4d effective picture" of the cyclic model, using the same ingredients as the inflationary model but with different details, not only assures skeptics, but is a powerful tool for analyzing the quantitative behavior of the cyclic model. Hence, no matter what your point of view, both the 5d brane picture and the 4d effective picture are useful for analyzing the cyclic model. For this reason, we will present both in this overview.

2.1 The Colliding brane picture

The cyclic theory of the universe is a direct outgrowth of an earlier cosmological model known as the "ekpyrotic" model.[Khoury et al.(2001)Khoury, Ovrut, Steinhardt, and Turok, Khoury et al.(2002a)Khoury, Ovrut, Steinhardt, an Khoury et al.(2002b)Khoury, Ovrut, Seiberg, Steinhardt, and Turok] The ekpyrotic model was motivated by the idea that the collision between two brane worlds approaching one another along an extra dimension could create a hot big bang from a nearly vacuous space-time before the collision. The name, ekpyrotic, meaning "out of the fire, is drawn from ancient cosmology of the Stoic philosophers in which the universe is created from the ashes of a fire and, after a long period of cooling, is consumed by fire again. The contemporary ekpyrotic concept is that the universe as we know it is made through a conflagration ignited by the collision of branes along a hidden fifth dimension.

The ekpyrotic model is related conceptually to the pre-big bang model developed several years earlier [Gasperini and Veneziano(1993), Gasperini et al.(1997)Gasperini, Maggiore, and Veneziano, Gasperini and Veneziano(2003)]. Both share the idea that the big bang is not the beginning of space and time and that the density perturbations responsible for galaxy formation were generated before the bang. However, the pre-big bang model does not include an ekpyrotic contraction phase with $w \gg 1$, which is crucial for smoothing and flattening the universe and for generating scale-invariant perturbations, as detailed in Sec. 3). Consequently, the pre-big bang model has no smoothing mechanism and generates an unacceptably blue spectrum of density perturbations (unless other components are added). In this sense, the ekpyrotic scenario is a significant advance.

The cyclic model differs from both the pre-big bang and ekpyrotic models because it is a comprehensive theory of the universe that incorporates the ekpyrotic theory of the big bang the dark energy phase observed today in a larger framework in which the two are intertwined. The big bang, instead of being a one-time event, repeats periodically every trillion years or so. This occurs through the regular collision of branes due to an interbrane force, plus the regular conversion of gravitational energy into brane kinetic energy and, ultimately, matter and radiation, as noted above.

In the cyclic theory, the inter-brane potential energy density at present corresponds to the currently observed dark energy, providing roughly 70% of the critical density today. The dark energy and its associated cosmic acceleration play a role in restoring the Universe to a nearly vacuous state thereby allowing the cyclic solution to become an attractor. The attractor behavior means that the cycling is stable. Random deviations from perfect cycling due to quantum or thermal effects are dissipated by the smoothing and red shift effect of dark energy so that the universe returns to the ideal cycling solution after a few cycles. Note that dark energy, which is unnecessary and added *ad hoc* in the big bang/inflationary picture is here an essential element associated with the fundamental force that drives the entire cyclic evolution.

In the colliding brane picture, the cyclic model assumes an overall geometry of two orbifolds planes, one with positive tension and one with negative tension, separated by an extra spatial dimension with negative cosmological constant, as suggested by heterotic M theory. The bulk space is warped, and the scale factor for each brane $a_{\pm}$ is purely a function of where the branes lie in the warped background. That is, when the branes are apart, $a_\pm$ have different values, and, when they collide, $a_+ = a_-$. In the absence of matter or an interbrane force, the configuration is in static equilibrium.

In the simplest treatment of the brane collision, the evolution is described solely by the time variation of the scale factors on the two branes, $a_\pm$, where the $\pm$ distinguishes the orbifold planes with positive and negative surface tension. Then these two degrees of freedom can be translated directly into 4d effective picture using the mapping

$$a_+ = 2\,a\cosh((\phi-\phi_\infty)/\sqrt{6}) \qquad a_- = -2\,a\sinh((\phi-\phi_\infty)/\sqrt{6}), \tag{1}$$

where a corresponds to the standard Robertson-Walker scale factor of the 4d effective theory and ϕ, known as the "radion," is an ordinary 4d scalar field.

These expressions are a dictionary for translating the 5d picture of colliding branes into a 4d effective picture of evolving quantum fields. The 4d radion field ϕ encodes information about the distance between branes in 5d: for example, the collision ($a_+ = a_-$ in the 5d picture) translates into $\phi \to -\infty$ in the 4d picture, and a finite gap corresponds to a finite value of ϕ. It is important to note that $a(t0$, the Friedmann-Robertson-Walker scale factor used to describe the expansion and contraction of the universe in the 4d effective picture, is neither the scale factor on positive tension brane (a_+) nor the negative tension brane (a_-), but rather $a = \frac{1}{2}\sqrt{a_+^2 - a_-^2}$. Consequently, even though the big crunch corresponds to a singularity ($a \to 0$) in the 4d coordinates, the same event in 5d coordinates corresponds to non-singular, finite values of $a_\pm$, whose evolution can be regular even when a vanishes. This feature is a sign of hope that the big bang singularity can translate into regular behavior in the 5d picture.

We have noted that the branes move in a 5d space with negative cosmological constant, an anti-de Sitter or AdS space. For branes in AdS, because a_+ and a_- are the scale factors on the positive and negative tension branes, the coupling to matter in the action is $a_\pm \rho_m^\pm$, where $\rho_m^\pm$ is the matter density on the positive (negative) tension brane. In the 4d effective coordinates, the coupling induces a interaction between matter and the radion field proportional to $\beta_+ = 2\cosh((\phi-\phi_\infty)/\sqrt{6}$ or $\beta_- = -2\sinh((\phi-\phi_\infty)/\sqrt{6})$, respectively.. (The constant field shift ϕ_∞ is arbitrary; it is convenient to choose $\phi = 0$ to be the zero of the potential $V(\phi)$.)

Since a_+ and a_- in low energy configurations are purely functions of their position in the AdS background and a_- corresponds to smaller warp factor than a_+, it is always the case that $a_- \leq a_+$, so that $a_+ = a_-$ is actually a boundary of moduli space. (The term “moduli refers to scalar degrees of freedom that determine the size and/or shape of extra dimensions.) One requires a matching rule to determine what the trajectory of the system does at that point. A natural matching rule is to suppose that at low energies and in the absence of potentials or matter, the branes simply bounce off one another with the intervening bulk briefly disappearing and then reappearing after collision.

The trajectory for the cyclic solution in the $a_+ - a_-$ plane is shown in Figure 2.1, obtained by solving the equations of motion including the effects of matter, radiation, and the interbrane potential energy.. The insert shows a blow-up of the behavior at the bounce in which the trajectory is light-like at contraction to the big crunch (the Universe is empty) and time-like on expansion from the big bang (radiation is produced at the bounce). In these coordinates, the scale factor increases exponentially over each cycle, but the next cycle is simply a rescaled version of the cycle before. A local observer measures physical quantities such as the Hubble constant or the deceleration parameter, which entail ratios of the scale factor and its derivatives in which the normalization of the scale factor cancels out. Hence, to local observers and in the 4d effective picture, each cycle appears to be identical to the one before.

2.2 The 4d Effective Field Theory Picture

The 4d effective field theory picture is often most convenient for describing the cyclic evolution away from the bounce, where the 4d effective scale factor a and the scalar (radion) field ϕ are non-singular and well-behaved. Then the cyclic model reduces to a four dimensional effective theory consisting of gravity coupled to one or more scalar fields.

Assuming the background universe is spatially flat, the metric is

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j \tag{2}$$

where $a(t)$ is the scale factor. The main imprint of the higher dimensional theory on the effective picture is through the addition of one or more scalar fields ϕ with a potential $V(\phi)$. This potential performs many functions in the cyclic model, including that of describing the dark energy responsible for the cosmic acceleration observed today. Except where otherwise noted, we shall describe the model in terms of a single scalar field, which suffices to describe many features of the model. (We will give an example where additional fields can play a role in the generation of a nearly scale invariant spectrum of density perturbations; see

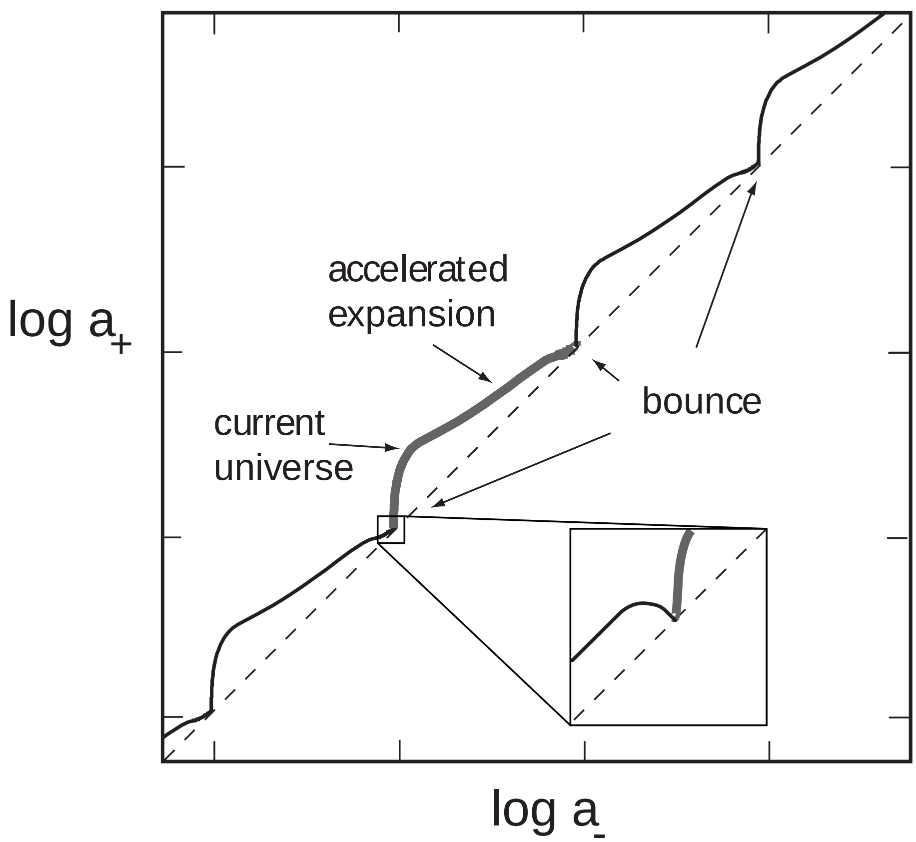

Figure 2: Schematic plot showing how the scale factors on the two branes, $a_{\pm}$, evolve over the course of many cycles (indicated by tick marks). A single cycle is indicated in bold. The dashed line represents the boundary $a_+ = a_-$ corresponding to a moment when the two branes collide and their scale factors are equal. Each cycle includes a brane kinetic energy, radiation, matter and dark energy dominated phase. Note that the evolution in this 5d picture is not really cyclic in the $a_{\pm}$ coordinates; each brane is stretching by an exponential factor (10^{120} or more) between bounces. However, 4d effective scale factor a and the radion field ϕ depend on combinations of $a_{\pm}$ that vary periodically from cycle to cycle. Hence, the evolution does appear cyclic when expressed in 4d effective coordinates.

Sec. 4). The scalar field ϕ satisfies

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}, \tag{3}$$

in the background (2), where dots denote derivatives with respect to t and $H \equiv \dot{a}/a$. Ignoring, for simplicity, the coupling between ordinary matter and ϕ, and the spatial curvature the Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{4}$$

G is Newtons constant and ρ is the energy density of ordinary matter and radiation.

The potential $V(\phi)$ is chosen by hand at present, but may be derivable from the higher dimensional theory [Steinhardt and Turok(2002a), Steinhardt and Turok(2002b), Buchbinder et al.(2007a)Buchbinder, Khoury, and Ovrut]. An example with the desired properties is

$$V(\phi) = V_0\left(e^{\alpha\phi} - e^{-\beta\phi}\right)F(\phi) \tag{5}$$

(see Fig. 3). Here V_0 is equal to today's dark energy density, α is non-negative (and typically $\ll 1$) and β is positive (and typically $\gg 1$). This ansatz for the V is motivated by string theory. When the branes are more than a few string lengths apart, the attractive force between them is expected to vary exponentially with the radion field ϕ with a coefficient that is itself ϕ-dependent. Here, for the purposes of illustration, we have approximated the ϕ-dependence in terms of two exponential terms in which the first dominates for large, positive ϕ and the second for negative ϕ. The factor $F(\phi)$ represents an effect that occurs when the branes are within a string lengthscale of one another. The extra dimension begins to disappear and a transition occurs in which the string coupling constant and all contributions to the potential fall rapidly to zero. For cosmology, the precise form of $F(\phi)$ is unimportant so long as it cuts off the steep exponential fall-off of the potential as ϕ moves from zero towards $-\infty$. In the example in Fig. (3), the steep decline cuts off near a negative minimum, denoted $V_{\rm end}$, where $\phi = \phi_{\rm end}$. The moment that V bottoms out, even if it were to remain non-zero, it ceases to affect the cosmological evolution for the remaining time before the crunch. This is because the scalar field kinetic energy, which was increasing and remaining comparable to $|V|$ during the ekpyrotic phase when V was steeply falling, continues to increase after V bottoms out. Consequently, the pressure and energy density both become kinetic energy dominated and their ratio $w \to 1$ independent of the precise form of F.

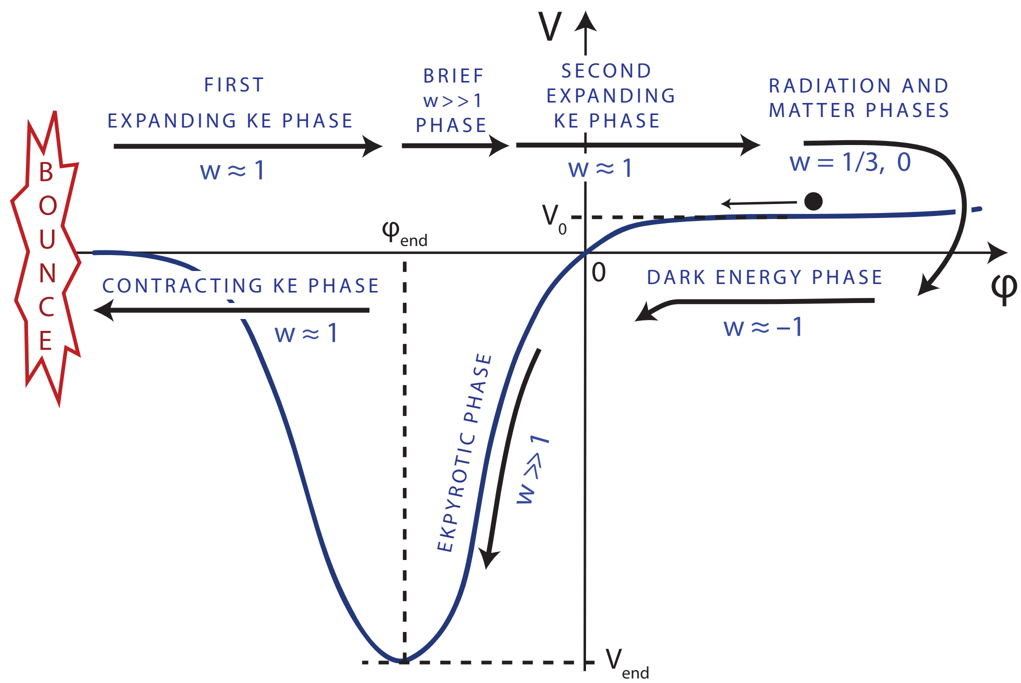

Figure 3: An example potential $V(\phi)$. This plot shows where ϕ is on its potential at each stage in a cycle. The ratio of pressure to energy density is denoted by w. V_0 is the potential energy density of the present dark energy phase. The future cycle evolution proceeds with ϕ moving from its present value (represent by the solid circle) towards the left, all the way to the bounce ($\phi \to -\infty$) and the back to the right through a sequence of short-duration phases with $w \approx 1$ or $w \gg 1$, and finally settling back at the value of ϕ it has today, after which the cycle begins anew. Over the next nine billion years, the universe undergoes the usual radiation and matter dominates phases, followed by a dark energy dominated phase with $w \approx= -1$. This phase ends when field ϕ begins to roll slowly towards $-\infty$ again and V becomes less than zero. Then begins the critical ekpyrotic contraction phase responsible for resolving the horizon and flatness problems and for generating a scale invariant spectrum of density perturbations; and it last until ϕ reaches ϕ_{end} at minimum of the potential $V = V_{end}$. After the ekpyrotic phase ends, the universe enters a scalar field kinetic energy dominated phase with $w \approx 1$ that endures all the way to the bounce.

Of central importance to the cyclic model is the *ekpyrotic* phase, in which the universe is slowly contracting and the scalar field is rolling slowly down its steeply declining, negative potential. (In the original ekpyrotic model, this corresponds to the motion of a bulk brane towards a boundary brane; in the cyclic model, it corresponds to the motion of two boundary branes towards one another.) The equation of state for the universe is:

$$w \equiv \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \tag{6}$$

which exceeds one when the potential energy density V is negative. For the sample potential, the ekpyrotic phase is the the period when the negative exponential dominates, $V(\phi) \approx -V_0 e^{-\beta\phi}$, and the background universe enters an attractor scaling solution,

$$a(t) \propto (-t)^{2/\beta^2} \propto e^{\phi/\beta}, \qquad H = \frac{2}{\beta^2 t} \propto -e^{-\beta\phi/2}, \qquad w \approx \beta^2/3 \gg 1, \tag{7}$$

in which t is negative and increasing. Notice that, since $\beta \gg 1$, the (4d effective) scale factor contracts very slowly compared to the Hubble radius $|H|^{-1}$. So, the situation is the inverse of inflation during which the scale factor grows much more rapidly than the Hubble radius.

The scaling solution is only relevant as long as $\phi > \phi_{\rm end}$, and the function $F(\phi)$ is effectively unity. Once ϕ passes ϕ_{end} and heads towards $-\infty$ (corresponding to the collision between branes), the potential energy is quickly converted to kinetic energy and the solution enters a kinetic energy dominated phase, with

$$a(t) \propto (-t)^{\frac{1}{3}} \propto e^{\phi/\sqrt{6}}, \qquad H = \frac{1}{3t} \propto -e^{-\sqrt{2/3}\phi}, \qquad w \approx 1. \tag{8}$$

This solution maintains itself all the way to $\phi \to -\infty$. The scalar field reaches this boundary at infinity in finite time and rebounds.

After the rebound, the solution followed is nearly the exact time-reverse of (8); the radiation and matter produced at the bang and a modest enhancement of the kinetic energy of ϕ have a negligible effect while $\phi < \phi_{\rm end}$. There is a brief $w \gg 1$ expanding phase right after ϕ passes $\phi_{\rm end}$ moving to positive values, but the excess kinetic energy in ϕ quickly overwhelms the potential energy $V(\phi)$ and the universe enters a second expanding kinetic phase (Figure 1). The expanding $w \gg 1$ phase is of modest duration and plays no significant role.

Continuing into the expanding phase, the kinetic energy in ϕ redshifts away as a^{-6} and the universe becomes dominated by the radiation that was produced at the bounce. The net expansion in the entire kinetic phase is $\sim e^{2\gamma/3}$, where $\gamma \equiv \ln((-V_{\rm end})^{1/4}/T_{\rm rh})$, and $T_{\rm rh}$ is by definition the temperature of the radiation when it comes to dominate. As shown in Ref. [Khoury et al.(2004)Khoury, Steinhardt, and Turok], cyclic models require $\gamma \sim 10-20$ in order to be compatible with observation. The additional Hubble damping due to the radiation has the effect of slowing ϕ down to a halt on the positive potential plateau. Then, the scalar field begins to gently roll downhill. The matter era passes and the universe enters the dark energy phase. Eventually, the rolling of ϕ carries it off the plateau and brings the universe back to the regime where $V < 0$. The accelerated expansion due to dark energy slows until the expansion halts altogether; then the slips immediately into a new phase of slow ekpyrotic contraction. The universe heads towards the next bounce and the next cosmic cycle.

Note that while the bounce itself is nearly symmetrical, the background evolution for $\phi > \phi_{\rm end}$ is highly asymmetrical, and the scale factor undergoes a large net expansion from cycle to cycle. As explained above, the kinetic phase gives a net expansion of $\frac{2}{3}\gamma$ e-folds. The ensuing radiation phase gives a large number of e-folds of expansion, and the matter phase adds a few more. We may approximate the combined number from the latter two phases as $N_{\rm rad} \equiv \ln(T_{\rm rh}/T_0)$, where T_0 is the cosmic microwave background temperature today. Dark energy adds another potentially large number of e-folds $N_{\rm dark}$. By contrast, in the ekpyrotic contraction phase, the scale factor contracts by a very modest factor (from Eq. 7, $a \propto H^{2/3w}$). The bottom line is: there is a large net expansion every cycle of approximately $2\gamma/3 + N_{\rm rad} + N_{\rm dark}$ e-folds. A timeline showing the evolution of the scale factor and the Hubble parameter during different epochs is shown in Fig. 2.2 The figure shows that the net expansion from one cycle to the next insures that perturbations from previous cycles, which pop outside the horizon during the dark energy and ekpyrotic phase, remain forever outside the horizon (because of the net expansion between cycles). The large net expansion also plays a key role in diluting the entropy density from cycle to cycle, and in the cyclic model's solution to the flatness, isotropy and horizon puzzles.

A very recent development is a variant known as the "new ekpyrotic model [Buchbinder et al.(2007a)Buchbinder, Khoury, a Buchbinder et al.(2007b)Buchbinder, Khoury, and Ovrut, Buchbinder et al.(2007c)Buchbinder, Khoury, and Ovrut, Creminelli and Senatore(2007)] in which the entire ekpyrotic scenario is described a 4d effective picture. The brane collision is replaced by a non-singular bounce (a reverses at some positive value) due to the introduction of ghost condensate fields. Ghost condensate fields have non-linear kinetic energy terms that can switch signs and create a period with $w < -1$, violating the weak positivity condition. Violation of weak positivity

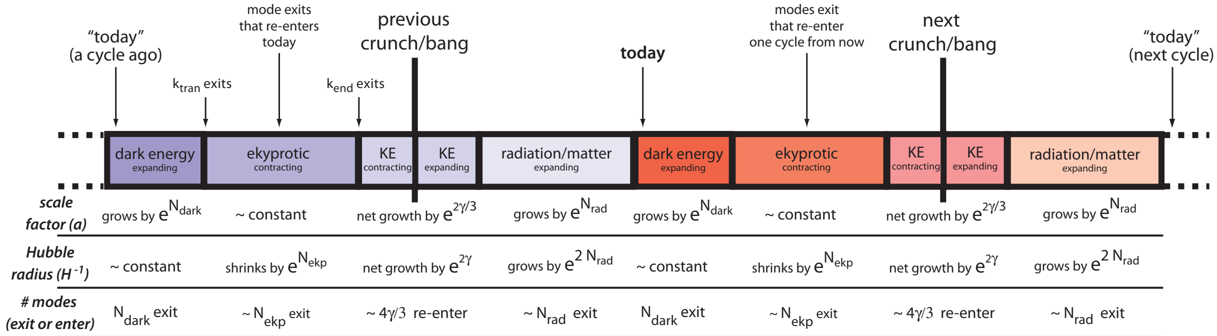

Figure 4: A timeline of the cyclic universe showing the behavior of key quantities before, during and after a given cycle, which may be taken to be the current one.. The labels "$k_{\rm tran}$ exits" and "$k_{\rm end}$ exits" indicate moments when modes in the density fluctuation spectrum at the transition from the dark energy to the ekpyrotic phase and at the end of the ekpyrotic phase, respectively. The modes within the observable horizon today exited during the last half the ekpyrotic phase preceding the most recent big bang.

is the only mechanism within Einsteins general theory of relativity that allows a to stop contracting at a finite value of a and transform smoothly to an expanding a. The same approach can be used, in principle, to construct a cyclic model with ordinary quantum fields. These initial examples have somewhat complicated constructions and the ghost condensate fields may prove to have some undesirable features, but the examples are significant because they show that, in principle, extra dimensions and branes and a singular big crunch/big bang transition are not required. Even for the colliding brane picture, they may also enable ordinary quantum field theory to be used to trace the entire cosmic evolution including the bounce (with an appropriate limiting process), a useful mathematical device for analyzing many problems.

3 The Ekpyrotic Phase

The critical and most novel aspect of the cyclic model is the period of contraction and bounce known as the "ekpyrotic phase." Previous attempts to construct oscillatory models all failed due to various problems that arise during a contraction phase: the matter and radiation density diverge; the entropy density diverges; the 4-curvature diverges; the anisotropy, spatial curvature, and inhomogeneity diverge; and collapse exhibits chaotic mixmaster behavior. This pathological behavior has rendered it inconceivable that a nearly homogeneous, isotropic and flat universe with small-amplitude scale-invariant fluctuations could emerge from a bounce. The cyclic model evades these problems because $w \gg 1$ during the ekpyrotic contraction phase, which changes the story entirely.

3.1 What is the $w > 1$ component?

The effects due to a $w > 1$ energy component are critical to the success of the cyclic scenario. Given earlier attempts at oscillatory models over the last century, one might naturally wonder why the introduction of a $w > 1$ phase was not considered previously. The probable reason is that, prior to inflation, cosmologists often assumed for simplicity that the universe is composed of "perfect fluids" for which $w = c_s^2$, where the equation of state w equals the ratio of pressure p to energy density ρ, and the speed of sound c_s is defined by $c_s^2 = dp/d\rho$. If $w > 1$ and the fluid is perfect, then $c_s > 1$, which is physically disallowed for any known fluid. With the advent of inflation, cosmologists have become more sophisticated and flexible about what fluids they are willing to consider. The inflaton, for example, has $w \approx -1$, yet the speed of sound is positive and well-behaved. A rolling scalar field with canonical kinetic energy has $c_s = 1$. Similarly, it is possible to have $w > 1$ and yet $0 \leq c_s^2 \leq 1$ without violating any known laws of physics. This opens the door to a novel kind of cyclic model.

The $w > 1$ equation of state derives naturally the interbrane potential that draws the branes together or, equivalently, the effective potential $V(\phi)$ for the radion scalar field.. The interbrane separation is described by a modulus field ϕ with an attractive potential that is positive when the branes are far apart and becomes negative as the branes approach. The rolling from a positive to a negative value is necessary for switching the universe from accelerated expansion to contraction. To see how this occurs, consider the Hubble parameter after the universe is dominated by the scalar field and its potential:

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V\right]. \tag{9}$$

The universe is spatially flat after a period of accelerated expansion, so we have included only the scalar field kinetic and potential energy density terms. In order to reverse from expansion to contraction, there must be some time when H hits zero. Since the scalar field kinetic energy density is positive definite, the only way H can be zero is if $V < 0$. So, reversal from accelerated expansion forces us to have V roll from a positive value (where V as the dark energy) to a negative value. However, $V < 0$ immediately implies an equation of state

$$w \equiv \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} > 1. \tag{10}$$

An example is steep part of our sample potential in Eq. (5) and Fig. 3, $V \propto -e^{-\beta\phi}$ with $\beta >> 1$, which draws ϕ towards an attractor solution with constant equation of state $w = (\beta^2 - 3)/3 \gg 1$. Hence, the

$w > 1$ component was, from the beginning, an essential component of the ekpyrotic phase needed to turn the universe from expansion to contraction; and, fortunately, achieving this equation of state simple to achieve with scalar fields and simple potentials. The real miracle, though, is that the same feature has two other consequences that were unanticipated when it was first introduced and that are essential for making the cyclic model viable.

3.2 Solving the homogeneity, isotropy and flatness problems without inflation

Before considering how an ekpyrotic contraction phase smooth and flatten the universe, it is useful to recall how an inflationary phase accomplishes the feat. In the standard big bang/inflation model, the universe is likely to emerge from the big bang with many ingredients contributing to the right hand side of the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\left[\frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\sigma^2}{a^6} + \ldots + \rho_I\right] - \frac{k}{a^2}, \tag{11}$$

where H is the Hubble parameter; a is the scale factor; $\rho_{m,r}$ is the matter and radiation density; σ^2 measures the anisotropy; k is the spatial curvature and ρ_I is the energy density associated with the inflaton. The parameters ρ_i and σ are constants which characterize the condition when $a = 1$, which we can choose without loss of generality to be the beginning of inflation. Each energy density term decreases as $1/a^{3(1+w)}$, for the value of w corresponding to that component. For the inflaton, we have assumed $w \approx -1$ and the energy density is nearly a-independent. The "..." refers to other possible energy components, such as the energy associated with inhomogeneous, spatially varying fields. Inflation works because all other contributions, including the spatial curvature and anisotropy, are shrinking rapidly as a grows, while the inflaton density ρ_I is nearly constant. Once the inflaton dominates, the future evolution is determined by its behavior alone and its decay products. The result is a homogeneous, isotropic and spatially flat universe.

Let's consider the same equation in a contracting universe. The term that will naturally dominate is the one that grows the fastest as a shrinks. In this case, the anisotropy term, σ^2/a^6 appears to out. A more careful analysis including the full Einstein equation reveals that the universe not only becomes anisotropic, but also develops a large anisotropic spatial curvature. This triggers "chaotic mixmaster behavior,"[Belinskii et al.(1970)Belinskii, Khalatnikov, and Lifshitz, Belinskii et al.(1973)Belinskii, Khalatnikov, and Lifshitz Demaret et al.(1986)Demaret, Hanquin, Henneaux, Spindel, and Taormina, Damour and Henneaux(2000)] resulting in unacceptably large inhomogeneity and anisotropy as the crunch approaches.

The story changes completely if there is an energy density component with $w > 1$ [Erickson et al.(2004)Erickson, Wesley, St
The brane/scalar field kinetic energy density decreases as

$$\frac{\rho_\phi}{a^{3(1+w)}}, \tag{12}$$

where the exponent $3(1+w) > 6$. Now, the scalar field density grows faster than the anisotropy or any other terms as the universe contracts. The longer the universe contracts, the more the scalar density dominates so that, by the bounce, the anisotropy and spatial curvature are completely negligible. Also negligible are spatial gradients of fields. The evolution is described by purely time-dependent factors, a situation referred to as *ultralocal.*

The striking discovery is that *a contracting universe with $w > 1$ has the same effect in homogenizing, isotropizing and flattening the universe as an expanding universe with $w < -1/3$.* This point is worth emphasizing, since some argued that the cyclic model relies on a dark energy phase to solve the horizon and flatness problems and, hence, should be regarded as just a variant of inflation [Kallosh et al.(2001)Kallosh, Kofman, and Linde, Linde(2002)]. It is now understood that dark energy is not needed for this purpose. In fact, if there were a period of dark energy expansion followed by a period of contraction with $w < 1$, the scenario would fail fail because, despite being rather homogeneous and flat at the end of the dark energy dominated phase, the universe would still enter the chaotic mixmaster phase before the crunch, leading to an unacceptable inhomogeneity, anisotropy and curvature.

3.3 $w > 1$ and scale-invariant perturbations

As shown above, having $w > 1$ during the ekpyrotic contraction phase makes the universe homogeneous and isotropic, classically. The same condition is responsible for generating a nearly scale-invariant spectrum of density perturbations when the effects of quantum fluctuations are included [Khoury et al.(2003)Khoury, Steinhardt, and Turo
Boyle et al.(2004)Boyle, Steinhardt, and Turok]. Here will treat the issue conceptually to make the comparison with inflation; a more detailed treatment is given in the next section.

Both the cyclic model and inflation generate density perturbations from sub-horizon scale quantum fluctuations. In each picture, there is one phase when the sub-horizon scale fluctuations exit the horizon and a much later phase when they re-enter.

In an exit phase with $\epsilon \equiv \frac{3}{2}(1+w)$, the scale factor $a(t)$ and the Hubble radius H^{-1} are related by the

Friedmann equations

$$a(t) \sim t^{1/\epsilon} \sim (H^{-1})^{1/\epsilon}. \tag{13}$$

If the universe is expanding during the exit phase, as in the case of inflation, quantum fluctuations leave the horizon if and only if a grows faster than H^{-1}. From the relation above, this requires $\epsilon < 1$ (or $w < -1/3$). A strong criterion must be satisfied if the spectrum is to be scale-invariant (spectral index $n_s \approx 1$): namely, H^{-1} must change very little during a period long enough for many modes to be stretched beyond the horizon. This corresponds to the limit $\epsilon \ll 1$. Then, the scalar spectral index n_s about some given wavenumber is [Khoury et al.(2003)Khoury, Steinhardt, and Turok, Boyle et al.(2004)Boyle, Steinhardt, and Turok]

$$n_s - 1 = -2\epsilon + \frac{d\, ln\, \epsilon}{dN} \tag{14}$$

where N is a time-like variable that measures the number of e-folds of inflation remaining when a given mode exits the horizon.

For a contracting universe, both the wavelength of the fluctuations and the Hubble radius are shrinking. In order for a mode to exit the horizon, the Hubble radius H^{-1} must Shrink *more rapidly* than the scale factor $a(t)$. According to (13), this requires $\epsilon > 1$ or $w > 1$. To obtain a spectrum that is nearly scale-invariant, we need a to be nearly constant over a period when H^{-1} changes a lot. This occurs if $\epsilon \gg 1$ or $w \gg 1$. This condition is generically obtained during the contraction phase of the cyclic model if the interbrane potential is negative and exponentially steep. Using standard methods, we obtain for the spectral index [Khoury et al.(2003)Khoury, Steinhardt, and Turok, Boyle et al.(2004)Boyle, Steinhardt, and Turok]

$$n_s - 1 = -\frac{2}{\epsilon} - \frac{d\, ln\, \epsilon}{dN}. \tag{15}$$

As ϵ (or w) increase, the spectrum is closer to scale-invariant.

Comparing (14) and (15), we see that the two expressions map into one another through the "duality transformation $\epsilon \to 1/\epsilon$. (The duality only applies to scalar perturbations at linear order.) This means that for each inflationary model with scalar spectral index n_s, there is a corresponding contracting (ekpyrotic) model with the same n_s. A corollary is that the Cyclic model and inflation cannot be distinguished by observing the (linear) scalar perturbations alone. We must dig further to determine if the perturbations were produced in a rapidly expanding phase or a slowly contracting phase. Some ways of distinguishing the two pictures are discussed in Sec. 4and 5.

4 Generating Density Perturbations during an Ekpyrotic Contraction Phase

This section discusses the generation of scalar (energy density) perturbations during an ekpyrotic contraction phase. Although the outcome is very similar to the density fluctuation spectrum for inflation, the mechanism is very different. In the inflationary picture, the physical mechanism for generating fluctuations relies on having a strong gravitational background in which de Sitter fluctuations excite the inflaton and all other fields with masses smaller than the Hubble parameter H during inflation. In the ekpyrotic picture, the generation mechanism is non-gravitational and only excites scale-invariant fluctuations in fields with steep potentials. To emphasize the point, we first show that scalar fields in Minkowski space with simple exponential potentials obtain a nearly scale-invariant spectrum of quantum fluctuations. Next, we discuss mechanisms for converting those scalar field fluctuations into curvature (density) perturbations. Finally, we discuss the leading non-gaussian contributions, which is an order of magnitude or more greater than those generated in the simplest inflationary models, a potentially testable prediction.

4.1 Scalar Field Perturbations in an Ekpyrotic Contraction Phase

We first consider ordinary scalar fields with exponentially steep potentials in the absence of gravity (Minkowski space) with action:

$$\mathcal{S} = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + V_0 e^{-\beta\phi} \right), \tag{16}$$

where a steep, negative exponential potential has been assumed. As the background scalar field rolls down the exponential potential towards $-\infty$, then, to leading order in $\hbar$, its quantum fluctuations acquire a scale-invariant spectrum, as the result of three features. First, the action (16) is classically scale-invariant. Second, by re-scaling $\phi \to \phi/\beta$ and re-defining V_0, the constant β can be brought out in front of the action and absorbed into Planck's constant $\hbar$, $\hbar \to \hbar/\beta^2$, in the expression $i\mathcal{S}/\hbar$ governing the quantum theory. Finally, it shall be important that ϕ has dimensions of mass in four spacetime dimensions.

To see the classical scale-invariance, note that shifting the field $\phi \to \phi + \epsilon$ and rescaling coordinates $x^\mu \to x^\mu e^{\beta\epsilon/2}$, just rescales the action by $e^{\beta\epsilon}$ and hence is a symmetry of the space of solutions of the classical field equations. As an initial condition for analyzing perturbations, it is reasonable to assume a spatially homogeneous background solution with zero energy density in the scalar field because the ekpyrotic phase is preceded by a very low energy density phase with an extended period of accelerated expansion, like that of today's universe, which drives the universe into a very low energy, homogeneous state. No new energy scale enters and the solution for the scalar field is then determined (up to a constant) by the scaling symmetry: $\phi_b = (2/\beta)\ln(-At)$. Next, consider quantum fluctuations $\delta\phi$ in this background. The classical equations are time-translation invariant, so a spatially homogeneous time-delay is an allowed perturbation, $\phi = (2/\beta)\log(-A(t+\delta t)) \rightarrow \delta\phi \propto t^{-1}$). On long wavelengths, for modes whose evolution is effectively frozen by causality, *i.e.* $|kt| \ll 1$, we can expect the perturbations to follow this behavior. Hence, the quantum variance in the scalar field,

$$\langle \delta\phi^2 \rangle \propto \hbar t^{-2} \tag{17}$$

Restoring β via $\phi \rightarrow \beta\phi$ and $\hbar \rightarrow \beta^2\hbar$ leaves the result unchanged. However, since $\delta\phi$ has the same dimensions as t^{-1} in four spacetime dimensions, it follows that the constant of proportionality in (17) is dimensionless, and therefore that $\delta\phi$ must have a scale-invariant spectrum of spatial fluctuations.

It is straightforward to check this in detail. Setting $\phi = \phi_b(t) + \delta\phi(t, \mathbf{x})$, to linear order in $\delta\phi$ the field equation is

$$\ddot{\delta\phi} = -V_{,\phi\phi}\delta\phi + \nabla^2\delta\phi, \tag{18}$$

and its solution as $|kt| \rightarrow 0$ is scale invariant

$$\langle \delta\phi^2 \rangle = \hbar \int \frac{k^2 dk}{4\pi^2} \frac{1}{k^3 t^2}, \tag{19}$$

where the modes are labeled by wavenumber k.

The generalization to two (or more) fields is straightforward. For example, consider two decoupled fields with a combined scalar potential

$$V_{tot} = -V_1 e^{-\int \beta_1 d\phi_1} - V_2 e^{-\int \beta_2 d\phi_2}, \tag{20}$$

where $\beta_1 = \beta_1(\phi_1)$, $\beta_2 = \beta_2(\phi_2)$, and V_1 and V_2 are positive constants. For the purposes of illustration, we focus on scaling background solutions in which both fields *simultaneously* diverge to $-\infty$. Then, as before, the fields each obtain nearly scale-invariant fluctuations; and, in particular, the relative fluctuation in the two fields, known as the "entropic perturbation

$$\delta s \equiv (\dot{\phi}_1 \delta\phi_2 - \dot{\phi}_2 \delta\phi_1)/\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2} \tag{21}$$

satisfies [Gordon et al.(2001)Gordon, Wands, Bassett, and Maartens]

$$\ddot{\delta s} + \left(k^2 + V_{ss} + 3\dot{\theta}^2\right)\delta s = 0, \tag{22}$$

where

$$V_{ss} = \frac{\dot{\phi_2}^2 V_{,\phi_1\phi_1} - 2\dot{\phi_1}\dot{\phi_2} V_{,\phi_1\phi_2} + \dot{\phi_1}^2 V_{,\phi_2\phi_2}}{\dot{\phi_1}^2 + \dot{\phi_2}^2},$$

$$\dot{\theta} = \frac{\dot{\phi_2} V_{,\phi_1} - \dot{\phi_1} V_{,\phi_2}}{\dot{\phi_1}^2 + \dot{\phi_2}^2}. \tag{23}$$

Here, $\dot{\theta}$ measures the bending of the trajectory in scalar field space, (ϕ_1, ϕ_2). For the simplest case, a straight line trajectory with $\dot{\theta} = 0$, the equation of motion (22) now becomes

$$\ddot{\delta s} + (k^2 + V_{,\phi\phi})\ \delta s = 0, \tag{24}$$

where we have set $\phi = \phi_1$. This is exactly the same equation as that governing the fluctuations of a single field (18), and so, since δs is a canonically normalized field according to its definition in (21), the power spectrum of δs generated from quantum fluctuations is scale-invariant.

Next we include gravity, so the action now becomes

$$\int d^4x \sqrt{-g} \big(\frac{1}{16\pi G} R - \frac{1}{2} \sum_{i=1}^{N} (\partial \phi_i)^2 - \sum_{i=1}^{N} V_i(\phi_i) \big). \tag{25}$$

Then, the entropy perturbation equation (22) in flat spacetime is replaced (see e.g. [Gordon et al.(2001)Gordon, Wands, Basse by

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2 \right) \delta s = \frac{4k^2 \dot{\theta}}{a^2 \sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}} \Phi \tag{26}$$

where Φ is the Newtonian potential. For the straight trajectory $\dot{\theta} = 0$, the entropy perturbation is not sourced by Φ and we can solve the equations rather simply; Ref. [Lehners et al.(2007a)Lehners, McFadden, Turok, and Steinhardt] for details. The result is a nearly scale-invariant spectrum with spectral index

$$n_s - 1 = \frac{2}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2}. \tag{27}$$

where

$$\epsilon \equiv \frac{3}{2}(1+w) \equiv \frac{\dot{\phi_1}^2 + \dot{\phi}_2^2}{2H^2} = \frac{\beta^2}{2(1+\gamma^2)}, \tag{28}$$

and $N \equiv \ln(a/a_{end})$ is the running parameter that measures the number of e-folds remaining in the ekpyrotic phase. This epitomizes the spectrum of entropic perturbations generated during the ekpyrotic contraction phase. We next turn to how the entropic perturbations are converted into curvature perturbations after the ekpyrotic phase is complete.

4.2 Conversion of Entropic to Curvature Perturbations

The generation of density perturbations in a contracting universe has been controversial due to a key difference in the nature of growing and decaying modes in a contracting phase compared to an expanding phase. In general, there are two independent modes, curvature fluctuations (on comoving hypersurfaces) and time-delay perturbations. In inflation, the curvature fluctuation on comoving hypersurfaces is the growing mode, and the time delay is a decaying mode; also, fluctuations of the inflaton excite directly the curvature mode. However, the roles are reversed in a contracting phase: curvature fluctuations shrink to zero, and the time-delay modes grow as the bounce approaches; fluctuations of the radion excite directly the time-delay mode, but this does not, by itself, create fluctuations in temperature and density. Instead, a mechanism is needed to convert time-delay modes into curvature modes just before or at the bounce.

The conversion is a significant issue. One of the theorems learned from studying perturbations in inflation is that the curvature fluctuation amplitude is conserved for modes outside the horizon [Bardeen(1980)]. If this were true for the cyclic model, then decaying curvature fluctuations before the bounce would imply negligible curvature fluctuations after the bounce (assuming no mode mixing at the bounce itself and the cyclic model would be inconsistent with observations. In particular, it can be shown that, if the cyclic picture could be reduced to a 4d effective field theory with a single scalar field (representing the radion), no method of conversion before the bound is known [Creminelli et al.(2005)Creminelli, Nicolis, and Zaldarriaga].

In the cyclic model, though, the 4d effective picture with one scalar field is only an approximation describing a few of the degrees of freedom of the 5d colliding brane picture. The branes and the bulk separating them make the critical difference in the conversion process.[Tolley et al.(2004)Tolley, Turok, and Steinhardt, Craps and Ovrut(2004), Battefeld et al.(2004)Battefeld, Patil, and Brandenberger] First, the branes define a precise hypersurface for the bounce from big crunch to big bang – the time-slice in which each point on one brane is in contact with a point on the other brane [Tolley et al.(2004)Tolley, Turok, and Steinhardt, Steinhardt and Turok(2005)]. Since ϕ is the modulus field that determines the bounce between branes, one might imagine that this corresponds to a surface with uniform $\delta\phi = -\infty$, which is a comoving hypersurface. However, ϕ measures the distance between branes only in the case that they are static. If the branes are moving, there are corrections to the distance relation due the excitation of bulk modes that depend on the brane speed and the bulk curvature scale [Tolley et al.(2004)Tolley, Turok, and Steinhardt]. Roughly speaking, the condition that $\delta\phi =$ be uniform ensures no velocity perturbations tangent to the branes, but simultaneous bounce requires no relative velocity perturbations perpendicular to the branes. Taking account of the gauge transformation required to transform from uniform $\delta\phi$ to the proper matching surface along which the collision is simultaneous introduces a scale-invariant curvature perturbation on the surface of collision. That is, in a collision-simultaneous gauge, the spatial metrics on the branes *necessarily* acquire long wavelength, nearly scale invariant curvature perturbations at the collision. This result has been obtained using several different methods by three independent groups [Tolley et al.(2004)Tolley, Turok, and Steinhardt, Craps and Ovrut(2004), Battefeld et al.(2004)Battefeld, Patil, and Brandenberger, McFadden et al.(2005)McFadden, Turok,

The lesson to learned from this computation is that the colliding brane picture does not reduce to a single scalar field degree of freedom plus gravity. If it did, then it would always be possible to choose a gauge where the scalar field is uniform and there is no curvature perturbations at all. In actuality, there are independent quantum fluctuations of both the radion and the bulk degrees of freedom that makes it impossible to treat the analysis in 4d effective theory. Analyzing the problem in 5d, one can show that it is possible to choose a gauge where one degree of freedom is spatially uniform, but not both. This means that there are always some local quantities that indicate non-uniformity. Then, it is possible to show that there is a mixing of growing and decaying modes near the big crunch/big bang transition, resulting in a nearly scale- invariant spectrum of curvature fluctuations before the bounce.

A demonstration of this effect in the 5d theory requires considerable mathematical technology [Tolley et al.(2004)Tolley, Tu McFadden et al.(2005)McFadden, Turok, and Steinhardt]. However, the same effect can be mimicked in a 4d effective theory by introducing two scalar fields that each obtain scale-invariant fluctuations in a contracting ekpyrotic phase, the example introduced in the previous subsection. Whereas one scalar field produces a pure time-delay mode, the second scalar field results in a second, scale-invariant "entropic mode that can be transformed into a curvature mode just before the bounce.

This entropic mechanism [Notari and Riotto(2002), Di Marco et al.(2003)Di Marco, Finelli, and Brandenberger, Lehners et al.(2007a)Lehners, McFadden, Turok, and Steinhardt] is an important advance both because it unambiguously demonstrates within context of a simple field theory that it is possible to produce a scale-invariant spectrum of curvature perturbations in a contracting phase and because it shows that extra dimensions and branes are not required to have an ekpyrotic or cyclic model. (Already several groups have developed variants of the ekpyrotic model based on the entropic mechanism and ordinary 4d quantum field theory

[Buchbinder et al.(2007a)Buchbinder, Khoury, and Ovrut, Buchbinder et al.(2007b)Buchbinder, Khoury, and Ovrut, Creminelli and Senatore(2007)].)

The entropic mode δs in Eq. (22) can be converted into a curvature perturbation if one or the other scalar fields undergoes a sudden acceleration. As an example, we will consider the common case where the scalar field trajectory encounters a boundary in moduli space and bounces off it. Such a bounce occurs in heterotic M-theory [Lehners et al.(2007b)Lehners, McFadden, and Turok, Lehners et al.(2007c)Lehners, McFadden, and Turok], when the negative tension brane bounces off the zero of the bulk warp factor just before the positive and negative tension branes collide. We refer to those papers for further details: for the purpose of this paper, all we need to know is that in the 4d effective description there are two scalar field moduli, ϕ_1 and ϕ_2, living on the half-plane $-\infty < \phi_1 < \infty$ and $-\infty < \phi_2 < 0$. Furthermore, the cosmological solution of interest is one in which ϕ_2 encounters the boundary $\phi_2 = 0$, and reflects off it elastically.

When the scalar field "reflects" off a boundary in moduli space, it converts entropy perturbations into curvature perturbations. As shown in Ref. [Gordon et al.(2001)Gordon, Wands, Bassett, and Maartens]), defining $\mathcal{R}$ to be the curvature perturbation on comoving spatial slices, for N scalar fields with general Kahler metric $g_{ij}(\phi)$ on scalar field space, the linearized Einstein-scalar field equations lead to

$$\dot{\mathcal{R}} = -\frac{H}{\dot{H}}\left(g_{ij}\frac{D^2\phi^i}{Dt^2}s^j - \frac{k^2}{a^2}\Psi\right), \tag{29}$$

where the $N-1$ entropy perturbations

$$s^i = \delta\phi^i - \dot{\phi}^i \frac{g_{jk}(\phi)\dot{\phi}^j\delta\phi^k}{g_{lm}(\phi)\dot{\phi}^l\dot{\phi}^m} \tag{30}$$

are just the components of $\delta\phi^i$ orthogonal to the background trajectory, and the operator D^2/Dt^2 is just the geodesic operator on scalar field space. In our case, things simplify because the scalar field space is flat, so the metric is $g_{ij} = \delta_{ij}$, and D/Dt reduces to an ordinary time derivative. For a straight line trajectory in field space, the right-hand side of (30) vanishes even if the entropy perturbation is nonzero. However, if there is a departure from geodesic motion, such as the reflection of ϕ_2 from the $\phi_2 = 0$ boundary of moduli space, the entropy perturbation directly sources the curvature perturbation.

4.3 Predictions for density fluctuations: amplitude, tilt and non- gaussianity

The curvature perturbation generated by converting entropic perturbations into curvature perturbations can be computed by integrating equation (29), as detailed in [Lehners and Steinhardt(2008a)]. In the example considered above, the conversion occurs after the ekpyrotic phase has ended ($t = t_{end}$) at time t_{ref} in the kinetic energy dominated phase when ϕ_2 reflects off the boundary of moduli space. The resulting curvature perturbation spectrum is [Lehners and Steinhardt(2008a)]

$$\langle \mathcal{R}^2 \rangle = \hbar \frac{\beta_1^2 |V_{end}|}{3\pi^2 M_{Pl}^2} \frac{\gamma^2}{(1+\gamma^2)^2} \left(1 + \ln(t_{end}/t_{ref})\right)^2 \int \frac{dk}{k} \equiv \int \frac{dk}{k} \Delta_{\mathcal{R}}^2(k) \tag{31}$$

for the perfectly scale-invariant case. Notice that the result depends only logarithmically on t_{ref}: the main dependence is on the minimum value of the effective potential V_{end} (see Fig. 3) and the parameter that determines the steepness of potential for ϕ_1, β_1. (The result depends on the steepness of the potentials for both ϕ_1 and ϕ_2, but the two are related by the condition $\dot{\phi}_2 = \gamma \dot{\phi}_1$. Since $\gamma = 1/\sqrt{3}$, $\beta_{1,2}$ are of the same order of magnitude; henceforth, we will drop the subscript $\beta \equiv \beta_1$.) Observations on the current Hubble horizon indicate $\Delta_{\mathcal{R}}^2(k) \approx 2.2 \times 10^{-9}$. Ignoring the logarithm in (31), this requires $\beta |V_{min}|^{\frac{1}{2}} \approx 10^{-3} M_{Pl}$, or approximately the GUT scale. This is entirely consistent with heterotic M-theory.

As for the spectral tilt, Eq. (27) becomes

$$n_s - 1 = \frac{4(1+\gamma^2)}{\beta^2 M_{Pl}^2} - \frac{4\beta_{,\phi}}{\beta^2}, \tag{32}$$

where we have used the fact that $\beta(\phi)$ has the dimensions of inverse mass. The presence of M_{Pl} clearly indicates that the first term on the right is a gravitational term. It is also the piece that makes a blue contribution to the spectral tilt. The second term is the non-gravitational term and agrees precisely with the flat space-time result.

For a pure exponential potential, which has $\beta_{,\phi} = 0$, the non-gravitational contribution is zero, and the spectrum is slightly blue, as our model-independent analysis suggested. For plausible values of $\beta = 20$ and $\gamma = 1/2$, say, the gravitational piece is about one percent and the spectral tilt is $n_s \approx 1.01$, also consistent with our earlier estimate. However, this case with $\beta_{,\phi}$ precisely equal to zero is unrealistic. In the cyclic model, for example, the steepness of the potential must decrease as the field rolls downhill in order that the ekpyrotic phase comes to an end, which corresponds to $\beta_{,\phi} > 0$. If $\beta(\phi)$ changes from some initial value $\bar{\beta} \gg 1$ to some value of order unity at the end of the ekpyrotic phase after ϕ changes by an amount $\Delta\phi$, then $\beta_{,\phi} \sim \bar{\beta}/\Delta\phi$. When β is large, the non-gravitational term in Eq. (32) typically dominates and the spectral tilt is a few per cent towards the red.

Our expression for the spectral tilt of the entropically induced curvature spectrum can also be expressed in terms of the customary "fast-roll" parameters [Gratton et al.(2004)Gratton, Khoury, Steinhardt, and Turok]

$$\bar{\epsilon} \equiv \left(\frac{V}{M_{Pl} V_{,\phi}}\right)^2 = \frac{1}{\beta^2} \qquad \bar{\eta} \equiv (\frac{V}{V_{,\phi}})_{,\phi}. \tag{33}$$

Note that $\bar{\epsilon} = 1/(2(1+\gamma^2)\epsilon)$. Then, the spectral tilt is

$$n_s - 1 = \frac{4(1+\gamma^2)}{M_{Pl}^2}\bar{\epsilon} - 4\bar{\eta}. \tag{34}$$

By comparison, the spectral tilt for inflation is

$$n_s - 1 = -6\epsilon + 2\eta \tag{35}$$

where the result is expressed in terms of the slow-roll parameters $\epsilon \equiv (1/2)(M_{Pl} V_{,\phi}/V)^2$ and $\eta \equiv V_{,\phi\phi} M_{Pl}^2$. Here we have revealed the factors of M_{Pl} to illustrate that both inflationary contributions are gravitational in origin. So, the range of spectral tilts for the simplest inflationary and ekpyrotic models are slightly different, with the ekpyrotic edging closer to $n_s = 1$, but there is also considerable overlap, especially when more general potentials are considered.

Finally, a distinctive prediction of ekpyrotic and cyclic models is that the density fluctuations are significantly more non-gaussian than inflationary models. A general density fluctuation spectrum can be characterized by a series of n-point correlation functions

$$\langle \rho(x_1)\rho(x_2)\ldots\rho(x_n)\rangle \tag{36}$$

where $\langle\ldots\rangle$ represents an average over all possible combinations of points x_1 thru x_n. The discussion above concerning scale-invariance and tilt referred specifically to the two-point function (also known as the power spectrum). If the spectrum is gaussian, all n-point fluctuations for odd n are zero and for even n are expressible as powers of the two-point function. Hence, all information in a gaussian spectrum is encoded in the two–point function. Both inflation and ekpyrotic/cyclic models predict a predominantly gaussian spectrum but also small non-gaussian contributions. These can be most easily detected by measuring the three-point function, also known as the bispectrum. And deviation from zero is a sign of non-gaussianity.

In principle, the bispectrum, which measures a deviation from non-gaussianity known as skewness, can take many functional forms depending on the source of the non-gaussianity. In both the inflationary and ekpyrotic/cyclic models, the non-gaussianity is due to non-linear evolution of scalar fields that varies from point to point, so-called 'local' non-gaussianity. In this case, the non-gaussianity can be expressed as a correction to the leading linear gaussian curvature perturbation, $\mathcal{R}_L$ which, following the notational convention used in [Langlois and Vernizzi(2007)], can be expressed as $\mathcal{R} = \mathcal{R}_L - \frac{3}{5} f_{NL} \mathcal{R}_L^2$, where the sign convention for the coefficient f_{NL} is the same as in Ref. [Komatsu and Spergel(2000)]. The parameter f_{NL} is nearly scale-invariant and can have either sign in principle. A positive sign corresponds to positive skewness in the matter distribution (more structure) and negative skewness in the CMB temperature fluctuations (more cold spots). The magnitude of f_{NL} is a measure of the degree of non-gaussianity. A key prediction of ekpyrotic/cyclic models is that f_{NL} is $\mathcal{O}(100)$ or more times the value predicted by inflationary models ($f_{NL} \lesssim .1$).[*]

Although the precise value of f_{NL} can vary significantly in both models, there is simple, intuitive reason why the non-gaussianity in ekpyrotic/cyclic models is generically several orders of magnitude greater than in inflationary models. The reason traces back to a defining feature that strongly distinguishes the two models: the difference in the equation of state during the period that density fluctuations are generated. In standard versions of both models, the density perturbation spectra have their origin in scalar fields ϕ_i which develop nearly scale invariant perturbations while evolving along an effective potential $V(\phi_i)$. However, the potential is nearly constant during an inflationary phase in order to obtain $w_{inf} \approx -1$ or, equivalently, $\varepsilon_{inf} \equiv \frac{3}{2}(1 + w_{inf}) \ll 1$; by contrast, the potential should be exponentially steep and negative to obtain $\varepsilon_{ek} \gg 1$, as required for an ekpyrotic phase. This means that the inflaton is nearly a free field with nearly gaussian quantum fluctuations. The non-gaussian amplitude depends on the deviation of the potential from perfect flatness or, equivalently, how close the slow-roll parameter ε_{inf} and its variation with time are to zero. This intuitive argument is consistent with the quantitative expression for f_{nL}obtained for inflationary models [Maldacena(2003)]. However, a steep potential means that the scalar fields in the ekpyrotic model necessarily have significant nonlinear self-interactions whose magnitude depends just how large ε_{ek} is. Because the magnitude of ε_{ek} is $\mathcal{O}(100)$ or more times larger than ε_{inf}, the scalar field contribution to the non-gaussianity – which we will call the "intrinsic" part – is correspondingly larger for ekpyrotic/cyclic models [Notari and Riotto(2002), Di Marco et al.(2003)Di Marco, Finelli, and Brandenberger, Buchbinder et al.(2007a)Buchbinder, Khoury, and Ovrut, Creminelli and Senatore(2007), Koyama et al.(2007)Koyama, Mizu Battefeld(2007), Lehners and Steinhardt(2008a)].

The magnitude of ε and its evolution during the last 60 e-folds of the inflationary or ekpyrotic/cyclic phase

[*]N.B. This statement, which comes from the abstract of Ref. [Lehners and Steinhardt(2008b)], refers to the intrinsic contribution to f_{NL}, as noted below, which is $\lesssim .1$ for inflation. Although the primary references by Lehners and the author state the condition correctly, a decimal point for the inflationary contribution was inadvertently missed in the earlier drafts of this review article. We regret any confusion this typo may have caused.

also determines the spectral tilt in the two models, as shown above. Hence, there is a natural correlation between n_s and non-gaussianity which makes the combined test more powerful than either individually [Lehners and Steinhardt(2008a)]. The fact that the observed spectral tilt only deviates by a few per cent from scale-invariant [Spergel et al.(2007)] tends to force w_{inf} to be closer to -1 and w_{ek} to be even greater, accentuating the difference in the predictions of the two models.

This intuitive argument only refers to the intrinsic contribution to the non-gaussianity, but this is enough to argue why ekpyrotic/cyclic models generically predict $|f_{NL}|$ to be several orders of magnitude greater than the value in inflationary models. First, the intrinsic contribution is the dominant contribution in simple examples of both models. Second, even if additional effects discussed below add or subtract from $|f_{NL}|$ in the ekpyrotic/cyclic model, obtaining a value less the one (that is, in the inflationary range) would only occur through accidental cancellations of independent terms at the two- or three-decimal level, which is highly unnatural. Conversely, it is possible to add features to inflationary models (such as curvatons, non-standard kinetic energy density, etc., with certain parameters) that enhance the non-gaussianity beyond $|f_{NL}| = 1$. These are unnecessary embellishments, though, and, when added, can produce virtually arbitrary f_{NL} of either sign.

The predictions can be further refined by taking account how the scalar field fluctuations are transformed into curvature perturbations. In inflationary models, the scalar field fluctuations directly produce curvature perturbations that are growing modes in an expanding universe. Consequently, the small intrinsic non-gaussianity in the scalar fields discussed above translates directly to small non-gaussianity in the curvature fluctuations. In ekpyrotic/cyclic models, the curvature perturbations produced directly by the scalar fields are decaying modes, as discussed above. Hence, the process involves scalar field fluctuations first producing growing mode entropic perturbations during the ekpyrotic phase and, then, converting them to curvature perturbations just before the bounce to an expanding phase [Notari and Riotto(2002), Di Marco et al.(2003)Di Marco, Finelli, and Brandenberger, Lehners et al.(2007a)Lehners, McFadden, Turok, and Steinhard
It is notable that the equation of state during this conversion (or ε_{conv}) can be quite different from the equation of state during the ekpyrotic phase (or ε_{ek}). Since the entire curvature perturbation is produced by this conversion, even the intrinsic contribution is necessarily affected by ε_{conv}, as well as ε_{ek}; in fact, the intrinsic f_{NL} turns out to be proportional to their geometric mean [Lehners and Steinhardt(2008b)], $\sqrt{\varepsilon_{ek}\varepsilon_{conv}}$. Hence, the magnitude of f_{NL} in ekpyrotic/cyclic models can be considerably less if the conver-

sion takes places in a kinetic energy dominated phase when $\varepsilon_{conv} = 3 \ll \varepsilon_{ek}.$, say, rather than the ekpyrotic phase [Lehners and Steinhardt(2008a)] (although both predict values much greater than the inflationary prediction).

The conversion mechanism can also determine the sign of the intrinsic contribution to f_{NL}. For example, the cyclic model discussed in Ref. [Lehners and Steinhardt(2008a)] produces a positive intrinsic f_{NL}, though other conversion mechanisms can produce either sign [Lehners and Steinhardt(2008b)]. Furthermore, the conversion from entropic to curvature perturbations necessarily introduces its own contribution to non-gaussianity that is small compared to the intrinsic contribution in some regimes but can be the dominant contribution in others. In the latter case, the net non-gaussianity tends to be so large that it is already ruled out by existing experiments. Hence, the empirically interesting regime appears to be the one in which the intrinsic contribution dominates. In this case, the predicted value of f_{NL} is $\mathcal{O}(1)\sqrt{\varepsilon_{ek}\varepsilon_{conv}}$ for ekpyrotic/cyclic models, where $\varepsilon_{ek} = \beta^2$. Hence, the prediction for ekpyrotic/cyclic models is

$$f_{NL} = \begin{cases} \beta, & \text{if conversion is in the KE dominated phase} \\ \beta^2. & \text{if conversion is in the ekpyrotic phase} \end{cases} \tag{37}$$

Since $\beta \gg 1$, this means $f_{NL} \gg 1$ compared with $|f_{NL}| < 1$ for inflationary models [Maldacena(2003)]. To be more quantitative, one can add the current observational measurement of the spectral index n_s, which suggests a slightly red tilt. As noted above, this requires a large β, roughly between 30 and 100. For the case where the conversion occurs during the kinetic energy dominated phase, the most natural possibility for the cyclic model, the prediction is roughly $f_{NL} \approx 30 - 100$. Although this may be the favored range, accounting for other contributions to non-gaussianity and variations of model-dependent parameters [Lehners and Steinhardt(2008a)] broadens the range to

$$-50 < f_{NL} < 200. \tag{38}$$

For the case of conversion during the ekpyrotic phase, the prediction is sseveral orders of magnitude higher.

The prediction is timely because this value is large enough to be detectable in current and near-future cosmic microwave background studies. The current bound reported by the WMAP collaboration, $-36 < f_{NL} < 100$ at 2σ [Spergel et al.(2007)] already disfavors the ekpyrotic case in which conversion occurs in the ekpyrotic phase. Intriguingly, a recent claimed detection of non-gaussianity $26.9 < f_{NL} < 146.7$ at 2σ [Yadav and Wandelt(2007)] based on a reanalysis of WMAP is inconsistent with simple inflationary models and right in line with the ekpyrotic/cyclic predictions if conversion takes place in the kinetic energy dominated phase, both in terms of sign and magnitude. The error will improve modestly with further WMAP data, and then dramatically ($\sigma \leq 5$) with the forthcoming Planck satellite mission and large scale structure studies.

So, even though a precise value of f_{NL} is not predicted, there is enough separation between the inflationary and ekpyrotic/cyclic predictions to make non-gaussianity a surprisingly good test. If observations of f_{NL} lie in the range predicted by the intrinsic contribution of either inflationary or ekpyrotic/cyclic models, it is reasonable to apply Occam's razor and Bayesian analysis to favor one cosmological model over the other. Combining with measurements of the spectral tilt significantly sharpens the test. The sign and magnitude of f_{NL} provides additional information about the equation of state when the curvature perturbations were generated and, in some cases, the type of inflationary or ekpyrotic/cyclic model. To learn more about the detailed calculations behind the prediction, the reader is referred to Ref. [Lehners and Steinhardt(2008b)], which derives analytic estimates of the contributions to non-gaussianity.

5 Primordial Gravitational Waves in the Cyclic Model

Although the spectra of density fluctuations are indistinguishable in ekpyrotic/cyclic and inflationary models to leading order, the spectra of gravitational waves to leading order are exponentially different. As a result, the search for primordial gravitational waves, either directly or their their imprint on the B-mode polarization of the cosmic microwave background radiation, is the most definitive way of distinguishing the two models.

The difference arises because the gravitational backgrounds are so dissimilar in the ekpyrotic and inflationary phases when the fluctuations are generated. During inflation, the Hubble parameter H is large and nearly constant, resulting in strong gravitational Fluctuations and a nearly scale-invariant spectrum. During an ekpyrotic phase, H is roughly comparable to the current Hubble parameter H_0, exponentially smaller than the case of inflation. Consequently, the gravitational effects are weak, and the primordial gravitational waves have an exponentially small amplitude. Also, H grows steadily throughout the ekpyrotic phase, which causes the gravitational wave spectrum to be blue, rather than scale-invariant.

The method for computing the spectrum is detailed in Ref. [Boyle and Steinhardt(2005)]. Here we review the final results: The gravitational wave spectrum for the cyclic model can be divided into three regimes. There is a low frequency (LF) regime corresponding to long wavelength modes that re-enter after matter-radiation equality (or wavenumbers $k < k_{eq}$), and a medium frequency (MF) regime consisting of modes which re-enter between equality and the onset of radiation domination ($k_{eq} < k < k_r$). (The dark energy dominated phase has a negligible effect.) The spectrum for the dimensionless strain for these two regimes is:

$$\Delta h \approx \frac{(k_{end}/k_r)^{\frac{1}{2}} k_0^2}{\pi M_{pl} H_r^{\alpha}} \begin{cases} k^{-1+\alpha} & (LF) \\ k^{\alpha}/k_{eq}, & (MF) \end{cases} \tag{39}$$

where k_0 labels the mode whose wavelength is equal to the present Hubble radius, the subscript r refers to quantities evaluated at the beginning of the radiation-dominated phase after the bounce, and $\alpha \equiv 2/(\beta^2 - 2) \ll 1$. The spectrum is blue with an amplitude that is exponentially small on horizon scales compared to the prediction for the simplest inflationary models. Finally, modes which exit the horizon during the ekpyrotic phase (before τ_{end}), and re-enter during the expanding kinetic phase (after the bound but before τ_r) result in a high frequency (HF) band ($k_r < k < k_{end}$):

$$\Delta h \approx \left(\frac{\sqrt{2}}{\pi}\right)^{\frac{3}{2}} \frac{(k_{end} H_r/k_r)^{\frac{1}{2}-\alpha} k_0^2}{M_{pl} k_{eq} k_r} \left|\cos\left(k\tau_r - \frac{\pi}{4}\right)\right| k^{\frac{1}{2}+\alpha} \quad (HF) \tag{40}$$

The spectrum can also be characterized in terms of $\Omega_{gw}(k, \tau_0)$, the fractional energy density in primordial gravitational waves (ρ_{gw})) per unit logarithmic wavenumber, in units of the critical density [Thorne(1987), Turner(1997)]:

$$\Omega_{gw}(k, \tau_0) \equiv \frac{k}{\rho_{cr}} \frac{d\rho_{gw}}{dk} = \frac{1}{6} \left(\frac{k}{k_0}\right)^2 \Delta h(k, \tau_0)^2 \; . \tag{41}$$

As shown in Fig. 5, $\Omega_{gw}(k, \tau_0)$ in the cyclic model is very blue, with nearly all the gravitational wave energy concentrated at the high-frequency end of the distribution. By contrast, the primordial spectrum of gravitational waves in inflation is nearly flat.

The calculation above shows that the primordial gravitational waves produced in the cyclic model are tens of orders below the detectable limit for a wide range of frequencies. An important correction to this prediction, though, is the indirect production of gravitational waves induced by the nearly scale-invariant spectrum of density perturbations.[Mollerach et al.(2004)Mollerach, Harari, and Matarrese, Ananda et al.(2007)Ananda, Clarkson, and Baumann et al.(2007b)Baumann, Steinhardt, Takahashi, and Ichiki] This effect is often ignored for inflationary models because it is small compared to the primordial gravitational wave contribution. However, for the cyclic model, this "scalar-induced contribution is actually the leading source of gravitational waves at long wavelengths. A comparison of this scalar-induced contribution to the primordial spectrum for typical inflationary model (with a tensor-to-scalar ratio $r = 0.1$) is shown in Fig. 5. The scalar-induced contribution only applies to direct measurements of gravitational waves, such as BBO. The scalar-induced spectrum grows with time so that, extrapolating back to the decoupling of the cosmic background radiation, the amplitude was too small to be detected. Hence, it left a completely negligible imprint

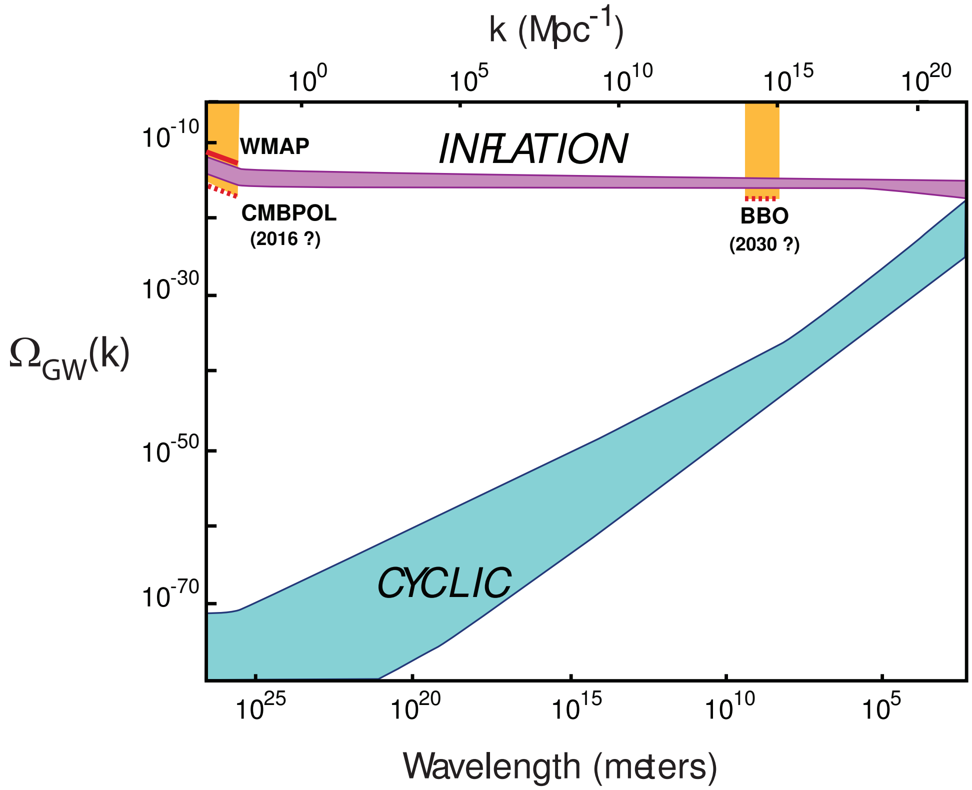

Figure 5: The fractional energy density in primordial gravitational waves per unit logarithmic wavelength in units of critical density for the cyclic and ekpyrotic models versus the wavelength of the gravitational wave. Superimposed are the current sensitivities for the Wilkinson Microwave Anisotropy Probe (WMAP) satellite and the sensitivities projected for future satellite missions to detect gravitational waves through their B-mode polarization effect on the cosmic microwave background (CMBPOL) and through direction detection in a space-based gravitational wave detector network such as the Big Bang Observatory (BBO).

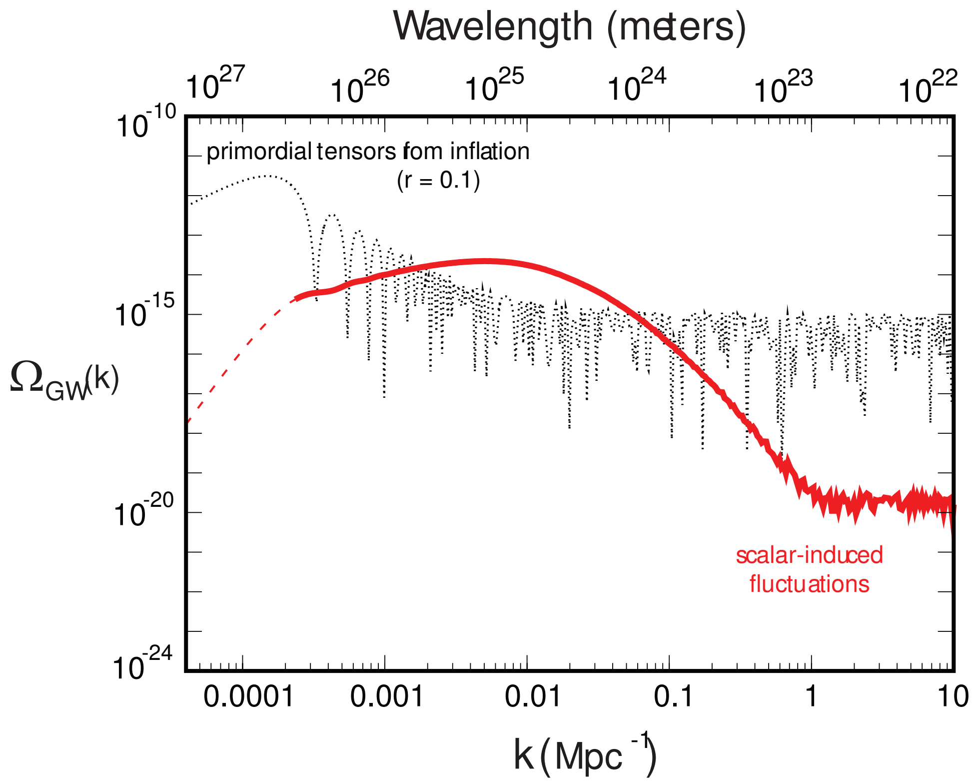

Figure 6: $\Omega_{gw}(k)$, the fractional energy density in primordial gravitational waves per unit logarithmic wavenumber, in units of critical density for a typical inflationary model versus the scalar-induced gravitational waves that occur in both inflationary and cyclic models. The wave number k is measured in inverse megaparsecs (1 Mpc $= 3\times10^{22}$ m) For the cyclic models, the scalar-induced contribution is always dominant, whereas this is only the case for inflationary models with a high degree of fine-tuning. The scalar-induced contribution only applies to direct measurements of gravitational waves, such as BBO, since the contribution is highly suppressed at high red shift.

on the cosmic microwave background polarization [Mollerach et al.(2004)Mollerach, Harari, and Matarrese, Ananda et al.(2007)Ananda, Clarkson, and Wands, Baumann et al.(2007b)Baumann, Steinhardt, Takahashi, and Ichiki].

6 The Cyclic Model is *not* a Perpetual Motion Machine.

The notion of an eternally cycling model raises the specter of a perpetual motion and violations of the laws of thermodynamics. In this section, we explain why the cyclic does not correspond to a perpetual motion machine of the first kind (violation of conservation of energy) or second kind (conversion of stored energy to kinetic energy without dissipation).

First, it is important to realize that the cyclic model is not truly periodic. The extra dimension expands and contracts at regular intervals, but the branes undergo a net stretching from cyclic to cycle; that is, $a_{\pm}$ are increasing from cycle to cycle, as shown in Fig. 2.1. Local observables, such as temperature and density, depend only on the expansion and contraction of the extra dimension; so they return to the same value once per cycle. Global quantities, like the total energy, total entropy, total number of black holes, and the cosmological constant, depend on the brane scale factors directly; so they vary monotonically. The facts that the branes are stretching from cycle to cycle and the total entropy and black hole increase over time are signs that there is more than the interbrane force providing energy to the system. Indeed, gravity is playing a critical role.

Gravity is the reason why the cyclic universe is not a perpetual motion machine of the first kind. Energy is conserved as the universe cycles even though the brane collisions are inelastic because gravity supplies extra energy during each contraction phase. During contraction, the kinetic energy of particles or, in this case, branes, is blue shifted due to gravity. So, when the branes collide, it is with greater energy than would be obtained with the interbrane force alone. Gravity provides the extra kinetic energy needed so that matter and radiation can be created at the collision, and the branes have sufficient kinetic energy remaining to return to their original positions. The cycles can repeat forever because gravity is a bottomless pit. In Newtonian gravity, for example, the gravitational potential energy is unbounded below. Similarly, in general relativity, there is no physical limit to how much energy can be drawn from the gravitational field. Because this regular (non-cyclic) creation of kinetic energy manifests only itself as the expansion of the branes and does not affect measurable quantities, like the matter density, temperature, and expansion rate, any local observer interprets the universe as being exactly cyclic. Behind the scenes, though, gravity is acting like an engine that keeps supplying more energy to keep the cycles going while respecting the conservation of energy.

The cyclic universe is not a perpetual motion machine of the second kind because it is not dissipationless [Steinhardt and Turok(2005)]. For example, black holes formed during one cycle will survive into the next cycle, acting as defects in an otherwise nearly uniform universe. Also, quantum fluctuations and thermal fluctuations will, with exponentially small rarity, create local regions which fall out of phase with the average cycling and possibly form giant black holes. In comoving coordinates, the black holes and bad regions increase in density over time, so the comoving observer sees the universe as "winding down." Similarly, a local observer will see the cycling as having finite duration in the sense that, at some point, after many, many cycles, he will end up inside a black hole (or bad region) and cease to cycle. Because of the stretching of space, the mean distance between the defective regions remains much larger than the Hubble distance. New cycling regions of space are being created although any one region of space cycles for a finite time. Thanks to gravity, which provides an eternal source of energy and continuously creates more space, the cyclic model satisfies the conventional thermodynamic laws even if the cycling continues forever.

Another suggestion has been that the holographic principle places a constraint on the duration of cycling [Steinhardt and Turok(2005)]. The argument is based on the fact that there is an average positive energy density per cycle. Averaging over many cycles, the cosmology can be viewed as an expanding de Sitter Universe. A de Sitter universe has a finite horizon with a maximal entropy within any observer's causal patch given by the surface area of the horizon. Each bounce produces a finite entropy density or, equivalently, a finite total entropy within an observer's horizon. Hence, the maximal entropy is reached after a finite number of bounces. (Quantitatively, a total entropy of 10^{90} is produced within an observer's horizon each cycle, and the maximum entropy within the horizon is 10^{120}, leading to a limit of 10^{30} bounces.)

Closer examination reveals a loophole in this analysis [Steinhardt and Turok(2005)]. Although the overall causal structure of the four-dimensional effective theory may be de Sitter, it is punctuated by bounces in which the scale factor approaches zero. See Figure 7. Each bounce corresponds to a spatially flat caustic surface. All known entropy bounds used in the holographic principle do not apply to surfaces which cross caustics. Holographic bounds can be found for regions of space between a pair of caustics (*i.e.*, within a single cycle), but there is no surface extending across two or more bounces for which a valid entropy bound applies. If the singular bounce is replaced by a non-singular bounce at a small but finite value of the scale factor, the same conclusion holds. In order for a contracting universe to bounce at a finite value of the scale factor, the null energy condition must be violated. However, the known entropy bounds require that the null energy condition be satisfied. Once again, we conclude that the entropy bounds cannot be extended across more than one cycle. Yet another way of approaching the issue is to note that both singular and non-singular bounces have the property that light rays focusing during the contracting phase defocus after the bounce, which violates a key condition required for entropy bounds. In particular, the light-sheet construction used in covariant entropy bounds [Bousso(2002)] are restricted to surfaces that are uniformly contracting, whereas the extension of a contracting light-sheet across a bounce turns into a volume with expanding area. Hence, if bounces are physically possible, entropy bounds do not place any restrictions on the number of bounces.

Does this mean that the cycling model has no beginning? This issue is not settled at present. Fig. 7 suggests that the cyclic model has the causal structure of an expanding de Sitter space. For de Sitter space,

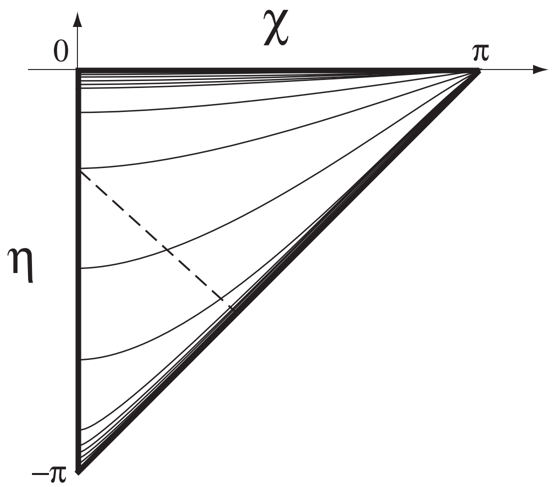

Figure 7: The diagram shows the conformal time, η, which can vary between $-\pi$ and zero, and the conformal spatial coordinate ξ, which varies between zero and π. The cyclic model has an average positive energy density per cycle, so its conformal diagram is similar to an expanding de Sitter space with constant density. The bounces occur along flat slices (curves) that, in this diagram, pile up near the diagonal and upper boundaries. For true de Sitter space, entropy bounds limit the total entropy in the entire spacetime. For the cyclic model, the bounds only limit the entropy between caustics (the bounces). A signal sent from the initial surface must travel through an infinite number of bounces to reach a current observer (dashed line).

the expanding phase is geodesically incomplete. In principle, physical particles can travel along unobstructed paths through an empty de Sitter space from before the initial expanding surface to the present in finite proper time. For inflation, this picture is used to explain that the de Sitter expansion cannot have been ongoing eternally in the past and that information about conditions prior to inflation can plausibly make its way to a present-day observer. One cannot avoid the question of what happened "before inflation."

For the cyclic model, the story is perhaps different. We have already seen that periodic bounces change the entropy bounds on the cyclic model compared to pure de Sitter space. Perhaps the bounces need to be considered here, as well. The analogous paths backwards in time travel through an unbounded number of bounces, each of which is a caustic surface with a high density of matter and radiation. Any particle attempting this trip will be scattered and likely annihilated before reaching a present-day observer. Consequently, no information may pass from the asymptotic past ($t \to -\infty$) to the present, and what we see today may be totally, or almost totally, insensitive to the initial state. Even though Fig. 7 suggests that the cyclic model is geodesically incomplete, there is, perhaps, no physical meaning to "before cycling" if observers receive all information from the cycling epoch which contains an infinite number of cycles. This issue is currently unresolved.

7 What we have learned and what we need to learn

What we have learned since the introduction of the cyclic theory of the universe six years ago has transformed it from a conceptual framework into a quantitative and predictive theory and has addressed most of the criticisms that have been raised. To appreciate the progress that has been made, it is worthwhile reviewing those criticisms and noting how the results described in the previous sections address them.

- *Does the cyclic model require fine-tuning of initial conditions?*
- *Is the cyclic model subject to chaotic mixmaster oscillations as the big crunch approaches, which generates unacceptable inhomogeneities and anisotropies?*
- *Does the cyclic model rely on dark energy and accelerated expansion to resolve the horizon and flatness problems; and, if so, should it be regarded as simply a variant of inflation?*

The answer to all these question is a definitive *no*, as is clear from the discussion in Sec. 3. The ekpyrotic contraction phase suffices to erase any inhomogeneity and anisotropy present as the dark energy dominated phase ends, even if the dark energy dominated phase lasts less than an e-fold [Erickson et al.(2004)Erickson, Wesley, Steinhardt, and Chaotic mixmaster behavior is completely suppressed because $w > 1$. And since ekpyrotic contraction (and not dark energy) is entirely responsible for resolving the horizon and flatness problems and for generating the scale-invariant spectrum of density fluctuations, the model is clearly distinct from inflation.

- *Can the cyclic model generate a nearly scale invariant spectrum of density perturbations before the big bang?*
- *Does the cyclic model predict any features in the density perturbation spectrum that differ from the simplest inflationary models?*
- *Does the cyclic model generate an observable spectrum of gravitational waves?*

The answer to these questions is a definitive *yes*, as delineated in Secs. 4 and 5. The issue of generating curvature perturbations, controversial originally, has been settled with the introduction of the entropic mechanism discussed in Sec. 4, which treats the generation of perturbations with ordinary scalar fields in 4d using concepts familiar from inflationary cosmology. Non-gaussianity and scalar-induced gravitational waves are signatures of this mechanism and prime observational targets for testing the cyclic model.

- *Does the cyclic model encounter the Tolman entropy problem that plagued earlier oscillatory models?*
- *Does the cyclic model conserve energy?*
- *Is the cyclic model a perfectly efficient engine?*
- *Does the holographic principle impose a limit on how many cycles the universe can have?*

The answer to these questions is *no*, as discussed in Sec. 6. With gravity as an eternal source of energy and as a driver of expansion, there can be unlimited cycles without violating any known principles (though any given finite volume of space cycles only a finite number of times).

What we still need to learn is how the assumed interbrane forces (or scalar field potentials) needed for the cyclic model arise from fundamental physics. The number of constraints is similar to inflationary cosmology, and some fully detailed examples have been constructed for ekpyrotic models [Buchbinder et al.(2007a)Buchbinder, Khoury, a that can be adapted to cyclic models. So, at this point, the issue is not considered to be a fundamental roadblock.

What we also need to learn is if it is possible to have a big crunch/big bang transition with the properties assumed in the cyclic theory. To deal with the big crunch/big bang, one has to go beyond Einsteins theory of gravity and describe space and time using concepts from quantum gravity. In the case of string theory, this means strings, branes, and perhaps the AdS/CFT (Anti-de Sitter/Conformal Field Theory) correspondence that relates gravitational phenomena to phenomena in conformal field theories without gravity.

As a first attempt, some [Turok et al.(2004)Turok, Perry, and Steinhardt] have considered a simplified treatment that focuses on the quantum production of matter and radiation in the last instants before the collision between branes. A natural concern is that the energy of a created particle becomes infinite as the separation between branes shrinks to zero; or, even if the energy of individual particles remains finite, their combined energy density could become infinite. Either effect might prevent the branes from bouncing apart and continuing to cycle. The big crunch/big bang transition has been investigated [Turok et al.(2004)Turok, Perry, and Steinhardt] for simplified brane collision in which the only excitations considered are membranes wrapped into tubes that connect the two branes and that form spontaneously through quantum fluctuations when the branes become very closely separated. As the separation between branes decreases, the tubes become short and squat and behave more and more like strings. Various oscillations of these strings are excited by the process, corresponding to the production of all possible quanta of the theory. The calculation shows that the membrane tubes evolve smoothly right up to and through the brane collision, revealing no obstacle to surviving the transition from big crunch to big bang. Furthermore, new tubes created as the two branes bounce apart snap and produce a finite density of hot particles and radiation on the two branes, in accord with the reheating process envisioned for the cycle picture.

There may be other corrections to Einstein gravity that are only relevant for a very short time (10^{-40} seconds) when the branes are within a string-length of one another and that are not captured by this simplified membrane model. However, there are reasons to be optimistic that they will not affect the overall picture. During this brief period, the modes of interest for cosmology, *e.g.,* wavelengths that lead to the formation of galaxies and larger scale structure, are far outside the horizon and their dynamics is frozen. Hence, it seems reasonable to match their linear behavior just before the bounce to the linear behavior just after the bounce without any significant alteration.

One approach to the matching problem is to avoid $t = 0$ altogether by analytically continuing in the complex t-plane in a semicircle with radius greater than the string scale and connecting negative to positive real values of t [Steinhardt and Turok(2005)]. Then, the linear analysis described above would remain essentially uncorrected by nonlinear gravitational effects, at least on long (three-dimensional) wavelengths. Work is currently in progress to construct such a continuation in nonlinear gravity.

Another approach is analyze the collision and bounce using the AdS/CFT correspondence. With the procedure, the big crunch/big bang transition in the gravitational theory (the bulk of AdS/CFT) maps precisely into a conformal theory without gravity (the surface of AdS/CFT). A recent attempt to apply this strategy [Turok et al.(2007)Turok, Craps, and Hertog] suggests a bounce is possible and that the matching conditions for the background evolution and the curvature perturbations agrees with the analytic continuation method described above and with the matching rules that assumed in the predictions of scalar and tensor perturbations in the previous sections.

For modes with wavelengths less than 10^{-30} cm, causal dynamics can alter the dynamics even within the final instants. In particular, the non-linear corrections to gravity could conceivably produce large amplitude effects that lead to the formation of many tiny primordial black holes. The black holes are problematic for those wishing to track precisely what occurs at the bounce. String theoretic methods are probably not powerful enough to analyze precisely this kind of inhomogeneous, non-linear regime. However, from a cosmological point-of-view, it is straightforward to envisage their effect, assuming that the branes bounce.

The black holes are small and have a mass sufficiently small that the black holes decay in much less than one second, well before primordial nucleosynthesis.

The black holes can be a boon to the scenario.[Baumann et al.(2007c)Baumann, Steinhardt, and Turok, Alexander and Meszaros(2007)] Their lifetime is long enough that they likely dominate the energy density before they decay. Consequently, their evaporation provides the entropy observed today. When they decay, their temperature rises near the end to values high enough to produce massive particles with baryon-number violating decays. At this point, the black holes are much hotter than the average temperature of the universe, so the decays occur when the universe is far from equilibrium. Assuming CP-violating interactions also exist, as in conventional high-temperature baryogenesis scenarios, the decay can produce the observed baryon asymmetry.

The big bang/big crunch transition is the leading concern of cosmologists regarding the cyclic theory. If this hurdle can be convincingly surmounted, then the cyclic model will be generally recognized as a theory that not unifies cosmology in an appealing way, but also gives a new perspective on time, allowing the possibility that time exists long before the big bang and perhaps infinitely many cycles into the past. Note, though, that the cyclic model is compatible with their being a beginning. One could imagine the sudden creation from nothing of two infinitesimal spherical branes arranged concentrically, both of which undergo continuous expansion and collision. Both would grow enormously with every new cosmic cycle. After several cycles, the branes would appear very flat and parallel to any observer. Since all traces of previous cycles is diluted exponentially over the course of one period, there would be very little difference between this universe with a beginning and a universe with two branes colliding eternally.

The emergence of the cyclic model as a viable competitor will also open minds to novel solutions to other theoretical issues, such as the cosmological constant (Λ) problem and the axion problem [Fox et al.(2004)Fox, Pierce, and Thor Steinhardt and Turok(2006)]. For example, the cyclic model enables a new approach for solving the cosmological constant problem in which Λ relaxes very slowly over course of many cycles from the Planck density to the exponentially smaller value it has today [Steinhardt and Turok(2006)]. The idea that Λ slowly relaxes has been tried within the context ofthe big bang/inflationary picture, but it is awkward to accommodate with inflation and dark energy. The relaxation must be very rapid during the first second of the universe to reduce Λ from the Planck scale to a level where it does not interfere with big bang cosmology, but this rapid reduction also tends to interfere with inflation; then, somehow, the relaxation must turn of at late times in order to allow for the current period of accelerated expansion. Introducing an awkwardly tuned relaxation mechanism to resolve the cosmological constant fine-tuning problem does not seem like progress.

The cyclic model introduces two important changes that free up constraints: (1) there is no inflation and, hence, no inflationary constraint on the relaxation rate; and, (2) the universe is much older, so it becomes to consider ultra-slow mechanisms for relaxing the cosmological constant with a characteristic timescale that is much greater then the Hubble time. is always far greater than the Hubble time. A possibility emerges in which the cosmological constant relaxes very slowly over the course of exponentially many cycles from a value near the Planck scale to the miniscule value observed today. One can then understand why the cosmological constant is exponentially smaller than one might guess based on the big bang picture; it is because the cosmological constant has had exponentially more time to relax than would be possible in the conventional big bang picture.

A specific example of an ultra-slow dynamical relaxation mechanism is an idea first proposed by Larry Abbott [Abbott(1985)] in the context of standard big bang cosmology (see alsoRef. ([Brown and Teitelboim(1987)])) in which the vacuum energy density of a scalar field gradually decays through a sequence of exponentially slow quantum tunneling events, relaxing an initially large positive cosmological constant to a small value. The proposal introduced an axion-like scalar field with a "washboard" potential

$$V(\phi) = -M^4 \cos\left(\frac{\phi}{f}\right) + \epsilon \frac{\phi}{2\pi f} + V_{other}, \tag{42}$$

where M is the axion mass scale; f is of order the Planck mass; $\epsilon \leq M^4$ is the coefficient of a linear term that breaks the shift symmetry ($\phi \to \phi + 2\pi$); and V_{other} incorporates all other contributions to the cosmological constant that one seeks to cancel. It is natural for axions to have an exponentially M and ϵ because both terms in V are created by exponentially suppressed, non-perturbative physics. The potential has a set of equally spaced minima V_N, with effective cosmological constants $\Lambda_{total} = 8\pi G V_N/3$ spaced by $8\pi G\epsilon/3$ (Fig. 8). No matter what value V_{other} has and no matter how large the initial vacuum density of the axion V_N, the universe can tunnel its way down the washboard from minimum to minimum, canceling the vacuum density coming from all other sources, until the net cosmological constant reaches the smallest possible positive value $\Lambda_{total} = 8\pi G V_0/3$.

Abbott assumed the universe emerges from the big bang and quickly settles into a minimum with large positive V_N, driving a period of inflation that dilutes any matter and radiation. Over time, the field ϕ

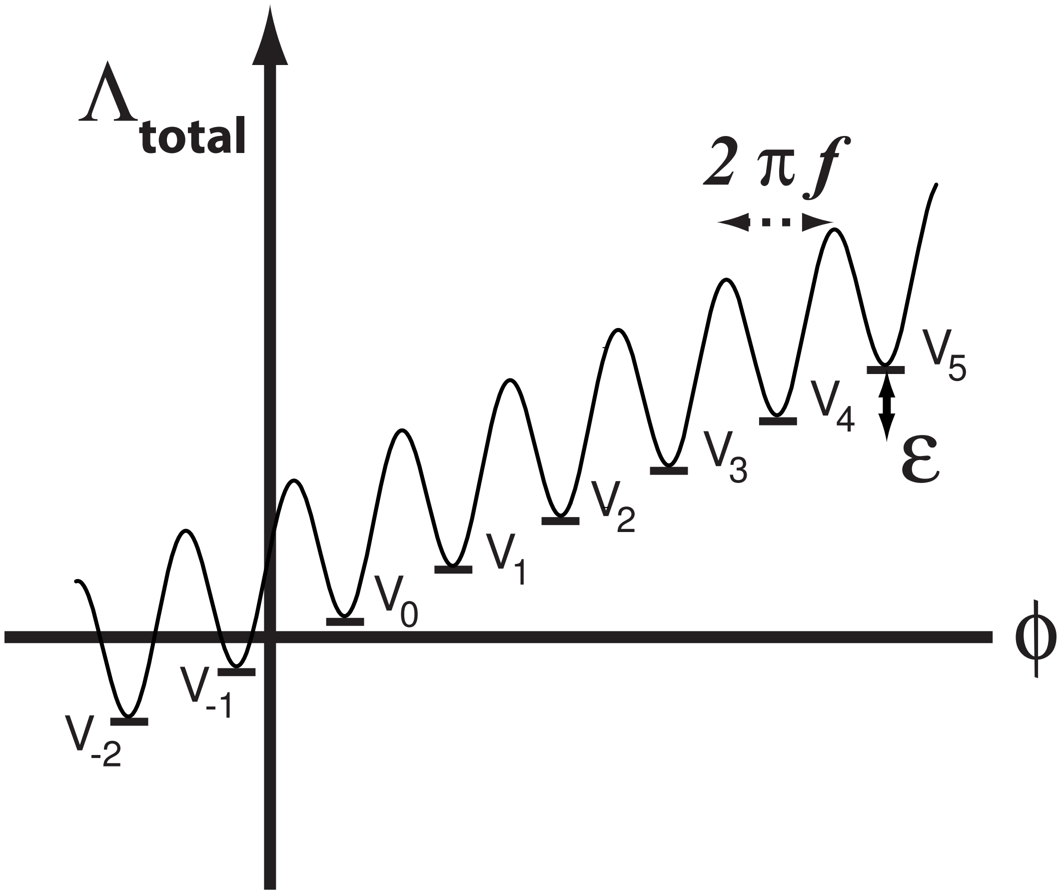

Figure 8: The effective cosmological constant for the washboard potential defined in Eq. 42 can take discrete values depending on which minimum V_N the axion-like scalar field ϕ occupies. The difference in energy density between two successive minima, $V_N - V_{N-1} = \epsilon$, is exponentially smaller than the Planck density. Beginning from a minimum with large positive cosmological constant, the universe tunnels progressively downhill, causing ϕ to change by $2\pi f$ and the cosmological constant to become smaller by ϵ. At each minimum of the washboard potential, the universe undergoes many cycles, and it spends exponentially more time in the minimum V_0 with the smallest positive cosmological constant than it does in all the other minima combined. Those regions that tunnel to a state with negative cosmological constant collapse within a Hubble time, but exponentially more space is created by repeated cycling than is lost.

works its way slowly but inexorably downhill. When the effects of gravity are included, the tunneling rate becomes slower and slower as V_N decreases. The universe remains in the last positive minimum for a relative eternity compared to the time spent in reaching it. Any specific region eventually tunnels to a minimum with negative V and collapses within a Hubble time; in the meantime, space remaining in the state with small positive cosmological constant expands by an exponential amount, leading to the successful prediction that the dominant volume of the universe has small positive cosmological constant.

The problem with Abbott's idea as originally posed is that the universe is completely empty by the time it tunnels even one step downhill. In essence, the relaxation takes so long that the matter density is too dilute by the time the universe reaches the state with small positive cosmological constant. This "empty universe" problem has never been solved within the context of inflationary cosmology.

With the cyclic model, though, it is possible to have the slow relaxation of the cosmological constant occur independently of the cycling and to have the big bangs the beginning of the cycles repeatedly replenish the universe with new matter and radiation. Here, as in the discussion of the entropy, the fact that the cyclic model is not truly periodic plays a crucial role. In particular, if the axion field is associated with one of the branes and the branes are expanding from cycle to cycle, it is possible to have the axions evolve steadily and the cosmological constant relax independently of the cycles of big bangs, reheating, and matter creation. (The axions are so weakly coupled that they are not affected by the reheating.) The only effec that the axion has on the cycles is that dark energy takes over at later stages in the cycle as the axion tunnels to lower minima of the potential. Each tunneling takes exponentially more tunneling than the ones before due to gravitational suppression effects [Steinhardt and Turok(2006)]. In particular, a given patch of the universe would survive $10^{10^{100}}$ cycles in the present state wth small positive cosmological constant before decaying to a state with negative cosmological constant and ceasing to cycle. The patches with negative cosmological constant would collapse within a Hubble time and not affect the overwhelming majority of space that continues to expand and cycle, since the decay rate is infinitesimal compared to the average expansion rate over the course of all those cycles. The resulting cosmology is in stark contrast to anthropic explanations according to which the only regions of space ever capable of producing galaxies, stars, planets and life are exponentially rare. With cycling and slow relaxation, an exponential majority of space-time is spent in a state with the small, positive cosmological constant.

One sure prediction is that the next few years will be an exciting period for the cyclic model as these theoretical issues will be aggressively pursued and experimental tests reach levels of sensitivity that can make or break the picture.

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